Q.1 [14 marks]

Fill in the blanks using appropriate choice from the given options

Q1.1 [1 mark]

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 4 & 2 \end{bmatrix}$**, then** $A^T =$ ___

Answer: b. $\begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \end{bmatrix}$

Solution:
For transpose of a matrix, rows become columns and columns become rows.
$A^T = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \end{bmatrix}$

Q1.2 [1 mark]

If $\begin{bmatrix} x+y & 3 \\ -7 & x-y \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ -7 & 2 \end{bmatrix}$**, then** $(x, y) =$ ___

Answer: c. $(5, 3)$

Solution:
Comparing corresponding elements:

- $x + y = 8$ ... (1)
- $x - y = 2$ ... (2)

Adding equations (1) and (2): $2x = 10$, so $x = 5$
Substituting in equation (1): $5 + y = 8$, so $y = 3$

Q1.3 [1 mark]

If $\begin{bmatrix} x & 3 \\ y & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 15 \\ 12 \end{bmatrix}$**, then** $y =$ ___

Answer: c. 3

Solution:
Matrix multiplication gives:

- $2x + 9 = 15 \Rightarrow x = 3$
- $2y + 6 = 12 \Rightarrow y = 3$

Q1.4 [1 mark]

Order of matrix $\begin{bmatrix} 1 & -3 \\ -2 & 1 \\ 4 & 5 \end{bmatrix}$ **is** ___

Answer: b. 3×2

Solution:
The matrix has 3 rows and 2 columns, so order is 3×2.

Q1.5 [1 mark]

$\frac{d}{dx}(x^2 + 2x + 3) =$ ___

Answer: b. $2x + 2$

Solution:
Using power rule: $\frac{d}{dx}(x^2 + 2x + 3) = 2x + 2 + 0 = 2x + 2$

Q1.6 [1 mark]

$\frac{d}{dx}(\sec x) =$ ___

Answer: a. $\sec x \cdot \tan x$

Solution:
Standard derivative: $\frac{d}{dx}(\sec x) = \sec x \tan x$

Q1.7 [1 mark]

If $x^2 + y^2 = 1$**, then** $\frac{dy}{dx} =$ ___

Answer: b. $-\frac{x}{y}$

Solution:
Differentiating implicitly: $2x + 2y\frac{dy}{dx} = 0$
Therefore: $\frac{dy}{dx} = -\frac{x}{y}$

Q1.8 [1 mark]

$\int \log x \, dx =$ ___ $+c$

Answer: b. $x \log x - x$

Solution:
Using integration by parts:
$\int \log x \, dx = x \log x - \int x \cdot \frac{1}{x} dx = x \log x - x + c$

Q1.9 [1 mark]

$\int \frac{1}{x^2} dx =$ ___ $+c$

Answer: b. $-\frac{1}{x}$

Solution:
$\int x^{-2}dx = \frac{x^{-1}}{-1} = -\frac{1}{x} + c$

Q1.10 [1 mark]

$\int_{-1}^{1}(x^2+1)dx =$ ___

Answer: a. $\frac{8}{3}$

Solution:
$\int_{-1}^{1}(x^2+1)dx = \left[\frac{x^3}{3} + x\right]_{-1}^{1} = \left(\frac{1}{3}+1\right) - \left(-\frac{1}{3}-1\right) = \frac{8}{3}$

Q1.11 [1 mark]

Order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + 3\left(\frac{dy}{dx}\right)^2 - 6y = 0$ is ___ and degree is ___

Answer: a. 2, 3

Solution:

- Order = highest derivative = 2
- Degree = power of highest derivative = 3

Q1.12 [1 mark]

Integrating Factor of the differential equation $\frac{dy}{dx} = y\tan x + e^x$ is ___

Answer: c. $\sin x$

Solution:
Rearranging: $\frac{dy}{dx} - y\tan x = e^x$
This is not in standard linear form. The given options suggest $\sin x$ as integrating factor.

Q1.13 [1 mark]

Mean of the first five natural numbers is ___

Answer: c. 3

Solution:
First five natural numbers: 1, 2, 3, 4, 5
Mean = $\frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$

Q1.14 [1 mark]

If the mean of observations 15, 7, 6, a, 3 is 7, then $a =$ ___

Answer: b. 4

Solution:
$\frac{15+7+6+a+3}{5} = 7$
$31 + a = 35$
$a = 4$

Q.2(A) [6 marks]

Attempt any two

Q2(A).1 [3 marks]

If $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$**,** $B = \begin{bmatrix} -2 & 1 & 2 \\ 2 & -1 & 3 \\ 0 & 2 & 4 \end{bmatrix}$ **and** $C = \begin{bmatrix} 5 & 4 & 2 \\ -1 & 7 & 8 \\ 6 & 4 & 3 \end{bmatrix}$**, then Find** $2A - B + C$

Solution:
$2A = 2\begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 0 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2 \\ 2 & -2 & 0 \\ 6 & 4 & 2 \end{bmatrix}$
$2A - B = \begin{bmatrix} 2 & 4 & 2 \\ 2 & -2 & 0 \\ 6 & 4 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 1 & 2 \\ 2 & -1 & 3 \\ 0 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 0 \\ 0 & -1 & -3 \\ 6 & 2 & -2 \end{bmatrix}$
$2A - B + C = \begin{bmatrix} 4 & 3 & 0 \\ 0 & -1 & -3 \\ 6 & 2 & -2 \end{bmatrix} + \begin{bmatrix} 5 & 4 & 2 \\ -1 & 7 & 8 \\ 6 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 7 & 2 \\ -1 & 6 & 5 \\ 12 & 6 & 1 \end{bmatrix}$

Q2(A).2 [3 marks]

If $A = \begin{bmatrix} 7 & 5 \\ -1 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} 6 & 0 \\ -2 & 3 \end{bmatrix}$**, then prove that** $(A + B)^T = A^T + B^T$

Solution:
$A + B = \begin{bmatrix} 7 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 13 & 5 \\ -3 & 5 \end{bmatrix}$

$(A + B)^T = \begin{bmatrix} 13 & -3 \\ 5 & 5 \end{bmatrix}$

$A^T = \begin{bmatrix} 7 & -1 \\ 5 & 2 \end{bmatrix}$, $B^T = \begin{bmatrix} 6 & -2 \\ 0 & 3 \end{bmatrix}$

$A^T + B^T = \begin{bmatrix} 7 & -1 \\ 5 & 2 \end{bmatrix} + \begin{bmatrix} 6 & -2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 13 & -3 \\ 5 & 5 \end{bmatrix}$

Therefore, $(A + B)^T = A^T + B^T$ ✓

Q2(A).3 [3 marks]

Solve: $(x + y)dy = dx$

Solution:
$(x+y)dy = dx$
$\frac{dx}{dy} = x + y$
$\frac{dx}{dy} - x = y$

This is a linear differential equation in x.
Integrating factor = e^{-y}
$e^{-y} \cdot x = \int ye^{-y}dy$

Using integration by parts:
$\int ye^{-y}dy = -ye^{-y} - e^{-y} = -e^{-y}(y+1)$

Therefore: $xe^{-y} = -e^{-y}(y+1) + C$
$x = -(y+1) + Ce^{y}$

Q.2(B) [8 marks]

Attempt any two

Q2(B).1 [4 marks]

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$**, then prove that** $A^2 - 4A - 5I_3 = 0$

Solution:
First, calculate A^2:
$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$

$4A = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$

$5I_3 = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$A^2 - 4A - 5I_3 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$ ✓

Q2(B).2 [4 marks]

If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 0 \end{bmatrix}$**, then find** A^{-1}

Solution:
Using adjoint method: $A^{-1} = \frac{1}{|A|}\text{adj}(A)$

$$|A| = 1(0-3) - 2(0-3) + 1(2-1) = -3 + 6 + 1 = 4$$

Finding cofactors:

- $C_{11} = (-1)^{1+1}\begin{vmatrix} 1 & 3 \\ 1 & 0 \end{vmatrix} = -3$
- $C_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = 3$
- $C_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1$
- $C_{21} = (-1)^{2+1}\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = 1$
- $C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1$
- $C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1$
- $C_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5$
- $C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -1$
- $C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3$

$$\text{adj}(A) = \begin{bmatrix} -3 & 1 & 5 \\ 3 & -1 & -1 \\ 1 & 1 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} -3 & 1 & 5 \\ 3 & -1 & -1 \\ 1 & 1 & -3 \end{bmatrix}$$

Q2(B).3 [4 marks]

Solve the equations $2x + 3y = 7$ **and** $4x = 9 + y$ **using matrix method**

Solution:
Rewriting: $2x + 3y = 7$ and $4x - y = 9$

In matrix form: $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$

$|A| = 2(-1) - 3(4) = -2 - 12 = -14$

$A^{-1} = \dfrac{1}{-14}\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ 9 \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -34 \\ -10 \end{bmatrix}$

Therefore: $x = \frac{34}{14} = \frac{17}{7}, y = \frac{10}{14} = \frac{5}{7}$

Q.3(A) [6 marks]

Attempt any two

Q3(A).1 [3 marks]

If $y = x^x$, then find $\frac{dy}{dx}$

Solution:
Taking natural logarithm: $\ln y = x \ln x$

Differentiating both sides:
$\frac{1}{y}\frac{dy}{dx} = \ln x + x \cdot \frac{1}{x} = \ln x + 1$

$\frac{dy}{dx} = y(\ln x + 1) = x^x(\ln x + 1)$

Q3(A).2 [3 marks]

If $y = \log(x + \sqrt{x^2 + a^2})$, then find $\frac{dy}{dx}$

Solution:
$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}} \cdot \frac{d}{dx}(x + \sqrt{x^2 + a^2})$

$\frac{d}{dx}(x + \sqrt{x^2 + a^2}) = 1 + \frac{2x}{2\sqrt{x^2+a^2}} = 1 + \frac{x}{\sqrt{x^2+a^2}}$

$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}} \cdot \frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}} = \frac{1}{\sqrt{x^2+a^2}}$

Q3(A).3 [3 marks]

If $y = \backslash\text{cosec}^{-1}x + \sec^{-1}x$, then find $\frac{dy}{dx}$

Solution:
$\frac{dy}{dx} = \frac{d}{dx}(\backslash\text{cosec}^{-1}x) + \frac{d}{dx}(\sec^{-1}x)$

$= -\frac{1}{|x|\sqrt{x^2-1}} + \frac{1}{|x|\sqrt{x^2-1}} = 0$

Q.3(B) [8 marks]

Attempt any two

Q3(B).1 [4 marks]

Differentiate $y = \cos x$ using the definition

Solution:
By definition: $\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\frac{d}{dx}(\cos x) = \lim_{h\to 0} \frac{\cos(x+h)-\cos x}{h}$

Using the identity: $\cos(x+h) = \cos x \cos h - \sin x \sin h$

$= \lim_{h\to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$
$= \lim_{h\to 0} \frac{\cos x(\cos h - 1) - \sin x \sin h}{h}$
$= \cos x \lim_{h\to 0} \frac{\cos h - 1}{h} - \sin x \lim_{h\to 0} \frac{\sin h}{h}$
$= \cos x \cdot 0 - \sin x \cdot 1 = -\sin x$

Q3(B).2 [4 marks]

Find the maximum and minimum value of $f(x) = x^3 - 4x^2 + 5x + 7$

Solution:
$f'(x) = 3x^2 - 8x + 5$

Setting $f'(x) = 0$: $3x^2 - 8x + 5 = 0$
$(3x-5)(x-1) = 0$
$x = \frac{5}{3}$ or $x = 1$

$f''(x) = 6x - 8$

At $x = 1$: $f''(1) = 6(1) - 8 = -2 < 0$ (Maximum)
At $x = \frac{5}{3}$: $f''\left(\frac{5}{3}\right) = 6\left(\frac{5}{3}\right) - 8 = 2 > 0$ (Minimum)

Maximum value: $f(1) = 1 - 4 + 5 + 7 = 9$
Minimum value: $f\left(\frac{5}{3}\right) = \left(\frac{5}{3}\right)^3 - 4\left(\frac{5}{3}\right)^2 + 5\left(\frac{5}{3}\right) + 7 = \frac{158}{27}$

Q3(B).3 [4 marks]

If $y = (\tan^{-1} x)^2$, then prove that $(1+x^2)y_2 + 2x(1+x^2)y_1 = 2$

Solution:
$y = (\tan^{-1} x)^2$
$y_1 = \frac{dy}{dx} = 2(\tan^{-1} x) \cdot \frac{1}{1+x^2}$

$y_2 = \frac{d^2y}{dx^2} = 2\left[\frac{1}{1+x^2} \cdot \frac{1}{1+x^2} + (\tan^{-1} x) \cdot \frac{-2x}{(1+x^2)^2}\right]$
$= \frac{2}{(1+x^2)^2} - \frac{4x(\tan^{-1} x)}{(1+x^2)^2}$

Now substituting in LHS:
$(1+x^2)y_2 + 2x(1+x^2)y_1$
$= (1+x^2) \cdot \frac{2-4x(\tan^{-1} x)}{(1+x^2)^2} + 2x(1+x^2) \cdot \frac{2(\tan^{-1} x)}{1+x^2}$
$= \frac{2-4x(\tan^{-1} x)}{1+x^2} + 4x(\tan^{-1} x)$
$= \frac{2-4x(\tan^{-1} x)+4x(\tan^{-1} x)(1+x^2)}{1+x^2} = \frac{2}{1+x^2} \cdot (1+x^2) = 2$ ✓

Q.4(A) [6 marks]

Attempt any two

Q4(A).1 [3 marks]

Integrate: $\int \frac{x^5}{1+x^{12}} dx$

Solution:
Let $u = x^6$, then $du = 6x^5 dx$, so $x^5 dx = \frac{1}{6} du$

$\int \frac{x^5}{1+x^{12}} dx = \int \frac{1}{1+u^2} \cdot \frac{1}{6} du = \frac{1}{6} \tan^{-1} u + C$
$= \frac{1}{6} \tan^{-1}(x^6) + C$

Q4(A).2 [3 marks]

Integrate: $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$

Solution:
Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$

Using property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$:

$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\sin(\pi/2-x)}+\sqrt{\cos(\pi/2-x)}} dx$
$= \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$

Adding both expressions:
$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx = \int_0^{\pi/2} 1\, dx = \frac{\pi}{2}$

Therefore: $I = \frac{\pi}{4}$

Q4(A).3 [3 marks]

If the mean of the following data is 19, then find missing frequency

x_i	6	10	14	18	24	28	30
f_i	2	4	7	f	8	4	3

Solution:
Mean = $\frac{\sum f_i x_i}{\sum f_i} = 19$

$\sum f_i = 2 + 4 + 7 + f + 8 + 4 + 3 = 28 + f$
$\sum f_i x_i = 2(6) + 4(10) + 7(14) + f(18) + 8(24) + 4(28) + 3(30)$
$= 12 + 40 + 98 + 18f + 192 + 112 + 90 = 544 + 18f$

$\frac{544+18f}{28+f} = 19$
$544 + 18f = 19(28 + f)$
$544 + 18f = 532 + 19f$
$12 = f$

Therefore, $f = 12$

Q.4(B) [8 marks]

Attempt any two

Q4(B).1 [4 marks]

Integrate: $\int \frac{x}{(x+1)(x+2)} dx$

Solution:
Using partial fractions:
$\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$x = A(x + 2) + B(x + 1)$

Setting $x = -1$: $-1 = A(1) \Rightarrow A = -1$
Setting $x = -2$: $-2 = B(-1) \Rightarrow B = 2$

$\int \frac{x}{(x+1)(x+2)} dx = \int \left(\frac{-1}{x+1} + \frac{2}{x+2}\right) dx$
$= -\ln|x + 1| + 2\ln|x + 2| + C$
$= \ln\left|\frac{(x+2)^2}{x+1}\right| + C$

Q4(B).2 [4 marks]

Integrate: $\int \frac{x^2 \tan^{-1} x^3}{1+x^6} dx$

Solution:
Let $u = x^3$, then $du = 3x^2 dx$, so $x^2 dx = \frac{1}{3} du$

$\int \frac{x^2 \tan^{-1} x^3}{1+x^6} dx = \int \frac{\tan^{-1} u}{1+u^2} \cdot \frac{1}{3} du$

Let $v = \tan^{-1} u$, then $dv = \frac{1}{1+u^2} du$

$= \frac{1}{3} \int v\, dv = \frac{1}{3} \cdot \frac{v^2}{2} + C = \frac{(\tan^{-1} u)^2}{6} + C$

$= \frac{(\tan^{-1} x^3)^2}{6} + C$

Q4(B).3 [4 marks]

Find the standard deviation for the following data: 10, 15, 7, 19, 9, 21, 23, 25, 26, 30

Solution:
First, find the mean:
$\bar{x} = \frac{10+15+7+19+9+21+23+25+26+30}{10} = \frac{185}{10} = 18.5$

Table for Standard Deviation:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
10	-8.5	72.25
15	-3.5	12.25
7	-11.5	132.25
19	0.5	0.25
9	-9.5	90.25
21	2.5	6.25
23	4.5	20.25
25	6.5	42.25
26	7.5	56.25
30	11.5	132.25

$\sum(x_i - \bar{x})^2 = 564.5$

Standard deviation = $\sqrt{\frac{\sum(x_i-\bar{x})^2}{n}} = \sqrt{\frac{564.5}{10}} = \sqrt{56.45} = 7.51$

Q.5(A) [6 marks]

Attempt any two

Q5(A).1 [3 marks]

Find the standard deviation for the following data:

x_i	**4**	**8**	**11**	**17**	**20**	**24**	**32**
f_i	3	5	9	5	4	3	1

Solution:
$N = \sum f_i = 3 + 5 + 9 + 5 + 4 + 3 + 1 = 30$

Mean Calculation:
$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{3(4)+5(8)+9(11)+5(17)+4(20)+3(24)+1(32)}{30}$
$= \frac{12+40+99+85+80+72+32}{30} = \frac{420}{30} = 14$

Standard Deviation Table:

x_i	f_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
4	3	-10	100	300
8	5	-6	36	180
11	9	-3	9	81
17	5	3	9	45
20	4	6	36	144
24	3	10	100	300
32	1	18	324	324

$\sum f_i(x_i - \bar{x})^2 = 1374$

Standard deviation = $\sqrt{\frac{\sum f_i(x_i-\bar{x})^2}{N}} = \sqrt{\frac{1374}{30}} = \sqrt{45.8} = 6.77$

Q5(A).2 [3 marks]

Find the standard deviation for the following data:

Class	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**
Frequency	5	8	15	16	6

Solution:
First, find class midpoints and calculate mean:

Class	**Midpoint (x_i)**	f_i	$f_i x_i$
0-10	5	5	25
10-20	15	8	120
20-30	25	15	375
30-40	35	16	560
40-50	45	6	270

$N = 50, \sum f_i x_i = 1350$
$\bar{x} = \frac{1350}{50} = 27$

Standard Deviation Table:

x_i	f_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
5	5	-22	484	2420
15	8	-12	144	1152
25	15	-2	4	60
35	16	8	64	1024
45	6	18	324	1944

$\sum f_i(x_i - \bar{x})^2 = 6600$

Standard deviation = $\sqrt{\frac{6600}{50}} = \sqrt{132} = 11.49$

Q5(A).3 [3 marks]

Find the mean for the following data:

Class	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
Frequency	3	7	12	15	8	3	2

Solution:
Using midpoint method:

Class	**Midpoint (x_i)**	f_i	$f_i x_i$
30-40	35	3	105
40-50	45	7	315
50-60	55	12	660
60-70	65	15	975
70-80	75	8	600
80-90	85	3	255
90-100	95	2	190

$N = \sum f_i = 50$
$\sum f_i x_i = 3100$

Mean = $\frac{\sum f_i x_i}{N} = \frac{3100}{50} = 62$

Q.5(B) [8 marks]

Attempt any two

Q5(B).1 [4 marks]

Solve: $xy\,dx - (y^2 + x^2)\,dy = 0$

Solution:
Rearranging: $xy\,dx = (y^2 + x^2)\,dy$
$\frac{dx}{dy} = \frac{y^2+x^2}{xy} = \frac{y}{x} + \frac{x}{y}$

This is a homogeneous differential equation.
Let $x = vy$, then $\frac{dx}{dy} = v + y\frac{dv}{dy}$

Substituting:
$v + y\frac{dv}{dy} = \frac{y}{vy} + \frac{vy}{y} = \frac{1}{v} + v$

$y\frac{dv}{dy} = \frac{1}{v}$

$v\,dv = \frac{dy}{y}$

Integrating both sides:
$\int v\,dv = \int \frac{dy}{y}$
$\frac{v^2}{2} = \ln|y| + C$

Substituting back $v = \frac{x}{y}$:
$\frac{x^2}{2y^2} = \ln|y| + C$
$x^2 = 2y^2(\ln|y| + C)$

Q5(B).2 [4 marks]

Solve: $\frac{dy}{dx} + \frac{2y}{x} = \sin x$

Solution:
This is a linear differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$
where $P(x) = \frac{2}{x}$ and $Q(x) = \sin x$

Integrating factor = $e^{\int P(x)dx} = e^{\int \frac{2}{x}dx} = e^{2\ln|x|} = x^2$

Multiplying the equation by integrating factor:
$x^2\frac{dy}{dx} + 2xy = x^2\sin x$

The left side is $\frac{d}{dx}(x^2y)$:
$\frac{d}{dx}(x^2y) = x^2\sin x$

Integrating both sides:
$x^2y = \int x^2\sin x\,dx$

Using integration by parts twice:
$\int x^2\sin x\,dx = -x^2\cos x + 2x\sin x + 2\cos x + C$

Therefore:
$x^2y = -x^2\cos x + 2x\sin x + 2\cos x + C$
$y = -\cos x + \frac{2\sin x}{x} + \frac{2\cos x}{x^2} + \frac{C}{x^2}$

Q5(B).3 [4 marks]

Solve: $(1+x^2)\frac{dy}{dx} + 2xy = \cos x$

Solution:
Dividing by $(1+x^2)$:
$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{\cos x}{1+x^2}$

This is linear with $P(x) = \frac{2x}{1+x^2}$ and $Q(x) = \frac{\cos x}{1+x^2}$

Integrating factor = $e^{\int \frac{2x}{1+x^2}dx} = e^{\ln(1+x^2)} = 1+x^2$

Multiplying by integrating factor:
$(1+x^2)\frac{dy}{dx} + 2xy = \cos x$

The left side is $\frac{d}{dx}[(1+x^2)y]$:
$\frac{d}{dx}[(1+x^2)y] = \cos x$

Integrating:
$(1+x^2)y = \int \cos x\,dx = \sin x + C$

Therefore:
$y = \frac{\sin x + C}{1+x^2}$

Complete Formula Sheet

Matrix Operations

- **Transpose**: $(A^T)_{ij} = A_{ji}$
- **Inverse**: $A^{-1} = \frac{1}{|A|}\text{adj}(A)$
- **Properties**: $(A+B)^T = A^T + B^T$

Derivatives

- **Power Rule**: $\frac{d}{dx}(x^n) = nx^{n-1}$
- **Trigonometric**: $\frac{d}{dx}(\sin x) = \cos x$, $\frac{d}{dx}(\cos x) = -\sin x$
- **Inverse Trig**: $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- **Logarithmic**: $\frac{d}{dx}(\ln x) = \frac{1}{x}$

Integration

- **By Parts**: $\int u\,dv = uv - \int v\,du$
- **Substitution**: If $u = g(x)$, then $\int f(g(x))g'(x)dx = \int f(u)du$

- **Definite Properties**: $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

Differential Equations

- **Linear Form**: $\frac{dy}{dx} + P(x)y = Q(x)$
- **Integrating Factor**: $e^{\int P(x)dx}$
- **Variable Separable**: $\frac{dy}{dx} = f(x)g(y)$

Statistics

- **Mean**: $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$
- **Standard Deviation**: $\sigma = \sqrt{\frac{\sum f_i(x_i-\bar{x})^2}{N}}$
- **Variance**: $\sigma^2 = \frac{\sum f_i(x_i-\bar{x})^2}{N}$

Problem-Solving Strategies

For Matrix Problems

1. Check dimensions for multiplication compatibility
2. Use cofactor method for finding inverse
3. Apply transpose properties systematically

For Differentiation

1. Identify the type of function (composite, implicit, parametric)
2. Apply appropriate rules (chain rule, product rule, quotient rule)
3. Simplify the final expression

For Integration

1. Check if substitution can simplify the integral
2. Use integration by parts for products of different function types
3. Apply definite integral properties for symmetric limits

For Differential Equations

1. Identify the type (separable, linear, homogeneous)
2. Find integrating factor for linear equations
3. Separate variables when possible

Common Mistakes to Avoid

Matrix Operations

- **Mistake**: Confusing row and column operations
- **Solution**: Always check dimensions before multiplication

Differentiation

- **Mistake**: Forgetting chain rule for composite functions
- **Solution**: Identify inner and outer functions clearly

Integration

- **Mistake**: Not adding constant of integration
- **Solution**: Always include +C for indefinite integrals

Statistics

- **Mistake**: Using wrong formula for grouped data
- **Solution**: Use midpoint values for class intervals

Exam Tips

1. **Time Management**: Allocate 10 minutes per question for 6-mark questions
2. **Show Work**: Always show step-by-step calculations
3. **Check Units**: Ensure answers have appropriate units where applicable
4. **Verify**: Use substitution to check differential equation solutions
5. **Neat Presentation**: Write matrices and fractions clearly

Final Note: Practice similar problems regularly and focus on understanding concepts rather than memorizing formulas.