Q.1 Fill in the blanks [14 marks]

Q1.1 [1 mark]

Order of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ -4 & 5 & 6 \end{bmatrix}$ is = __

Answer: (b) 2×3

Solution:
A matrix with 2 rows and 3 columns has order 2×3.

Q1.2 [1 mark]

If $\begin{bmatrix} x-3 & 2 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 4 & 0 \end{bmatrix}$ then x =

Answer: (d) 8

Solution:
For matrix equality, corresponding elements must be equal:
$x - 3 = 5$
$x = 8$

Q1.3 [1 mark]

The adjoint of $\begin{bmatrix} -3 & 2 \\ 0 & 1 \end{bmatrix}$ = ____

Answer: (b) $\begin{bmatrix} 1 & -2 \\ 0 & -3 \end{bmatrix}$

Solution:
For matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
$\text{adj} \begin{bmatrix} -3 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & -3 \end{bmatrix}$

Q1.4 [1 mark]

For any square matrix A, $(A^{-1})^{-1}$ = ___

Answer: (b) A

Solution:
By definition of inverse matrices: $(A^{-1})^{-1} = A$

Q1.5 [1 mark]

$\frac{d}{dx} \log x$ **= ____**

Answer: (b) $\frac{1}{x}$

Solution:
The derivative of natural logarithm: $\frac{d}{dx}\log x = \frac{1}{x}$

Q1.6 [1 mark]

$\frac{d}{dx}(\tan^{-1} x + \cot^{-1} x)$ = __

Answer: (d) 0

Solution:
$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ (constant)
Therefore, $\frac{d}{dx}(\tan^{-1} x + \cot^{-1} x) = 0$

Q1.7 [1 mark]

If $x = a\cos\theta$, $y = a\sin\theta$ then $\frac{dy}{dx}$ = _

Answer: (a) $-\cot\theta$

Solution:
$\frac{dx}{d\theta} = -a\sin\theta$, $\frac{dy}{d\theta} = a\cos\theta$
$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\cos\theta}{-a\sin\theta} = -\cot\theta$

Q1.8 [1 mark]

$\int 5x^4 dx$ = ___ + c

Answer: (d) x^5

Solution:
$\int 5x^4 dx = 5 \cdot \frac{x^5}{5} = x^5 + c$

Q1.9 [1 mark]

$\int_0^1 e^x dx$ = _

Answer: (a) $e - 1$

Solution:
$\int_0^1 e^x dx = [e^x]_0^1 = e^1 - e^0 = e - 1$

Q1.10 [1 mark]

$\int_{-1}^1 3x^2 - 2x + 1dx$ = _

Answer: (c) 4

Solution:
$\int_{-1}^1 (3x^2 - 2x + 1)dx = [x^3 - x^2 + x]_{-1}^1$
$= (1 - 1 + 1) - (-1 - 1 - 1) = 1 - (-3) = 4$

Q1.11 [1 mark]

The order of differential equation $(\frac{dy}{dx})^2 + 4y = x$ is __

Answer: (d) 1

Solution:
Order is the highest derivative present. Here, only first derivative $\frac{dy}{dx}$ appears, so order = 1.

Q1.12 [1 mark]

The integrating factor of $\frac{dy}{dx} + 3y = x$ is ____

Answer: (d) e^{3x}

Solution:
For linear DE $\frac{dy}{dx} + Py = Q$, integrating factor = $e^{\int Pdx}$
Here $P = 3$, so I.F. = $e^{\int 3dx} = e^{3x}$

Q1.13 [1 mark]

The mean of first ten natural numbers is____

Answer: (a) 5.5

Solution:
Mean = $\frac{1+2+3+...+10}{10} = \frac{55}{10} = 5.5$

Q1.14 [1 mark]

The range of the data 17, 15, 25, 34, 32 is ______

Answer: (d) 19

Solution:
Range = Maximum - Minimum = 34 - 15 = 19

Q.2 (A) Attempt any two [6 marks]

Q2.1 [3 marks]

If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ then find $A + A^T + I$.

Answer:

Solution:

$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A + A^T + I = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 1 \\ 1 & 7 \end{bmatrix}$

Q2.2 [3 marks]

If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ **then prove that** $A^2 - 4A + 7I_2 = 0$

Answer: Proved

Solution:
$A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$

$4A = 4 \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix}$

$7I_2 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$A^2 - 4A + 7I_2 = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$ ✓

Q2.3 [3 marks]

Solve differential equation $dy - 3x^2 e^{-y} dx = 0$

Answer: $e^y = x^3 + C$

Solution:
$dy - 3x^2 e^{-y} dx = 0$
$dy = 3x^2 e^{-y} dx$
$e^y dy = 3x^2 dx$

Integrating both sides:
$\int e^y dy = \int 3x^2 dx$
$e^y = x^3 + C$

Q.2 (B) Attempt any two [8 marks]

Q2.1 [4 marks]

Find the inverse of matrix $\begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$

Answer: $A^{-1} = \begin{bmatrix} 1/14 & 1/14 & -1/14 \\ -9/14 & -7/14 & 11/14 \\ -5/14 & -5/14 & 1/2 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$

First, find $\det(A)$:

$\det(A) = 3(1 \cdot 1 - (-1) \cdot 0) - (-1)(4 \cdot 1 - (-1) \cdot 5) + 2(4 \cdot 0 - 1 \cdot 5)$

$= 3(1) + 1(9) + 2(-5) = 3 + 9 - 10 = 2$

Since $\det(A) \neq 0$, inverse exists.

Finding cofactors and adjoint matrix:

$C_{11} = 1, C_{12} = -9, C_{13} = -5$

$C_{21} = 1, C_{22} = -7, C_{23} = -5$

$C_{31} = -1, C_{32} = 11, C_{33} = 7$

$$\text{adj}(A) = \begin{bmatrix} 1 & 1 & -1 \\ -9 & -7 & 11 \\ -5 & -5 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \cdot \text{adj}(A) = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 \\ -9 & -7 & 11 \\ -5 & -5 & 7 \end{bmatrix}$$

Q2.2 [4 marks]

If $A + B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ **and** $A - B = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$ **then find** AB**.**

Answer: $AB = \begin{bmatrix} 0 & -1 \\ 4 & -2 \end{bmatrix}$

Solution:

Adding the equations:

$(A + B) + (A - B) = 2A$

$2A = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 4 & 4 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$

Subtracting the equations:
$(A+B)-(A-B)=2B$
$2B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 2 & -4 \end{bmatrix}$
$B = \begin{bmatrix} -1 & -1 \\ 1 & -2 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 0 & -6 \end{bmatrix}$

Q2.3 [4 marks]

Solve the system of linear equation $2x+3y=1$, $y-4x=2$ using matrices.

Answer: $x=-\frac{1}{11}, y=\frac{13}{11}$

Solution:
The system can be written as: $AX=B$
$\begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\det(A)=2(1)-3(-4)=2+12=14$

$A^{-1} = \frac{1}{14}\begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$

$X = A^{-1}B = \frac{1}{14}\begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{1}{14}\begin{bmatrix} -5 \\ 8 \end{bmatrix}$

Therefore: $x=-\frac{5}{14}, y=\frac{8}{14}=\frac{4}{7}$

Q.3 (A) Attempt any two [6 marks]

Q3.1 [3 marks]

Find the derivative of $f(x)=e^x$ using definition of derivative.

Answer: $f'(x)=e^x$

Solution:
Using the definition: $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$f'(x)=\lim_{h\to 0}\frac{e^{x+h}-e^x}{h}$
$=\lim_{h\to 0}\frac{e^x\cdot e^h-e^x}{h}$
$=e^x\lim_{h\to 0}\frac{e^h-1}{h}$
$=e^x\cdot 1=e^x$

Q3.2 [3 marks]

If $\sqrt{x}+\sqrt{y}=\sqrt{a}$ then prove that $\frac{dy}{dx}=-\sqrt{\frac{y}{x}}$

Answer: Proved

Solution:
$\sqrt{x} + \sqrt{y} = \sqrt{a}$

Differentiating both sides with respect to x:
$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$

$\frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}} = -\sqrt{\frac{y}{x}}$ ✓

Q3.3 [3 marks]

Evaluate $\int \frac{\tan x}{\sec x + \tan x} dx$

Answer: $x - \ln|\sec x + \tan x| + C$

Solution:
Let $I = \int \frac{\tan x}{\sec x + \tan x} dx$

Multiply numerator and denominator by $(\sec x - \tan x)$:
$I = \int \frac{\tan x(\sec x - \tan x)}{(\sec x + \tan x)(\sec x - \tan x)} dx$
$= \int \frac{\tan x(\sec x - \tan x)}{\sec^2 x - \tan^2 x} dx$
$= \int \frac{\tan x(\sec x - \tan x)}{1} dx$
$= \int (\tan x \sec x - \tan^2 x) dx$
$= \int \tan x \sec x dx - \int (\sec^2 x - 1) dx$
$= \sec x - \tan x + x + C$

Q.3 (B) Attempt any two [8 marks]

Q3.1 [4 marks]

If $e^x + e^y = e^{x+y}$ **then find** $\frac{dy}{dx}$**.**

Answer: $\frac{dy}{dx} = \frac{e^x(e^y - 1)}{e^y(e^x - 1)}$

Solution:
$e^x + e^y = e^{x+y}$

Differentiating both sides with respect to x:
$e^x + e^y \frac{dy}{dx} = e^{x+y}(1 + \frac{dy}{dx})$
$e^x + e^y \frac{dy}{dx} = e^{x+y} + e^{x+y} \frac{dy}{dx}$

Rearranging:
$e^x - e^{x+y} = e^{x+y} \frac{dy}{dx} - e^y \frac{dy}{dx}$
$e^x - e^{x+y} = \frac{dy}{dx}(e^{x+y} - e^y)$

$\frac{dy}{dx} = \frac{e^x - e^{x+y}}{e^{x+y} - e^y} = \frac{e^x(1 - e^y)}{e^y(e^x - 1)} = \frac{e^x(e^y - 1)}{e^y(e^x - 1)}$

Q3.2 [4 marks]

For $y = 2e^{3x} + 3e^{-2x}$, prove that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$.

Answer: Proved

Solution:
$y = 2e^{3x} + 3e^{-2x}$

$\frac{dy}{dx} = 6e^{3x} - 6e^{-2x}$

$\frac{d^2y}{dx^2} = 18e^{3x} + 12e^{-2x}$

Now checking the equation:
$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y$
$= (18e^{3x} + 12e^{-2x}) - (6e^{3x} - 6e^{-2x}) - 6(2e^{3x} + 3e^{-2x})$
$= 18e^{3x} + 12e^{-2x} - 6e^{3x} + 6e^{-2x} - 12e^{3x} - 18e^{-2x}$
$= (18 - 6 - 12)e^{3x} + (12 + 6 - 18)e^{-2x}$
$= 0 \cdot e^{3x} + 0 \cdot e^{-2x} = 0$ ✓

Q3.3 [4 marks]

Equation of motion of a moving particle given by $s = t^3 + 3t$, $t > 0$, when the velocity and acceleration will be equal?

Answer: At $t = 1$ second

Solution:
Given: $s = t^3 + 3t$

Velocity: $v = \frac{ds}{dt} = 3t^2 + 3$
Acceleration: $a = \frac{dv}{dt} = 6t$

For velocity = acceleration:
$3t^2 + 3 = 6t$
$3t^2 - 6t + 3 = 0$
$t^2 - 2t + 1 = 0$
$(t - 1)^2 = 0$
$t = 1$

Therefore, velocity and acceleration are equal at $t = 1$ second.

Q.4 (A) Attempt any two [6 marks]

Q4.1 [3 marks]

Evaluate: $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

Answer: $-2\cos\sqrt{x} + C$

Solution:
Let $u = \sqrt{x}$, then $du = \frac{1}{2\sqrt{x}} dx$, so $dx = 2\sqrt{x} du = 2u du$

$\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx = \int \frac{\sin u}{u} \cdot 2udu = 2\int \sin udu = -2\cos u + C = -2\cos\sqrt{x} + C$

Q4.2 [3 marks]

Evaluate: $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$

Answer: $\frac{\pi}{4}$

Solution:
Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$

Using property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$:
$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}} dx$
$= \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$

Adding both expressions:
$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx = \int_0^{\pi/2} 1dx = \frac{\pi}{2}$

Therefore: $I = \frac{\pi}{4}$

Q4.3 [3 marks]

Find the mean of the frequency distribution:

Age	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59
Staff	5	7	9	11	10	8	6	4

Answer: Mean = 37.5 years

Solution:

Class	Midpoint (x)	Frequency (f)	fx
20-24	22	5	110
25-29	27	7	189
30-34	32	9	288
35-39	37	11	407
40-44	42	10	420
45-49	47	8	376
50-54	52	6	312
55-59	57	4	228
Total		**60**	**2330**

Mean = $\frac{\sum fx}{\sum f} = \frac{2330}{60} = 38.83$ years

---

Q.4 (B) Attempt any two [8 marks]

Q4.1 [4 marks]

Evaluate: $\int_0^1 \frac{x^2}{1+x^6} dx$

Answer: $\frac{\pi}{12}$

Solution:
Let $u = x^3$, then $du = 3x^2 dx$, so $x^2 dx = \frac{1}{3} du$
When $x = 0, u = 0$; when $x = 1, u = 1$

$\int_0^1 \frac{x^2}{1+x^6} dx = \int_0^1 \frac{1}{1+u^2} \cdot \frac{1}{3} du = \frac{1}{3} \int_0^1 \frac{1}{1+u^2} du$
$= \frac{1}{3}[\tan^{-1} u]_0^1 = \frac{1}{3}(\tan^{-1} 1 - \tan^{-1} 0) = \frac{1}{3} \cdot \frac{\pi}{4} = \frac{\pi}{12}$

Q4.2 [4 marks]

Find area enclosed by curve $y = x^2$, X-axis and $x = 2$

Answer: Area = $\frac{8}{3}$ square units

Solution:
The area is bounded by $y = x^2$, $y = 0$ (X-axis), $x = 0$ and $x = 2$

Area = $\int_0^2 x^2 dx = \left[\frac{x^3}{3}\right]_0^2 = \frac{8}{3} - 0 = \frac{8}{3}$ square units

Q4.3 [4 marks]

Calculate the standard deviation for the following continuous grouped data:

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

Answer: Standard deviation = 10.95

Solution:

Class	Midpoint (x)	f	fx	x^2	fx^2
0-10	5	5	25	25	125
10-20	15	8	120	225	1800
20-30	25	15	375	625	9375
30-40	35	16	560	1225	19600
40-50	45	6	270	2025	12150
Total		**50**	**1350**		**43050**

Mean $\bar{x} = \frac{1350}{50} = 27$

Variance = $\frac{\sum fx^2}{n} - (\bar{x})^2 = \frac{43050}{50} - (27)^2 = 861 - 729 = 132$

Standard deviation = $\sqrt{132} = 11.49$

Q.5 (A) Attempt any two [6 marks]

Q5.1 [3 marks]

If mean of 25 observation is 50 and mean of other 75 observation is 60. Considering all the observation then find the mean.

Answer: Combined mean = 57.5

Solution:
Combined mean = $\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}$
$= \frac{25\times50+75\times60}{25+75} = \frac{1250+4500}{100} = \frac{5750}{100} = 57.5$

Q5.2 [3 marks]

Find the mean deviation for the following frequency distribution:

x_i	3	4	5	6	7	8
f_i	1	3	7	5	2	2

Answer: Mean deviation = 1.1

Solution:

x_i	f_i	$f_i x_i$	$\|x_i - \bar{x}\|$	$f_i\|x_i - \bar{x}\|$
3	1	3	2	2
4	3	12	1	3
5	7	35	0	0
6	5	30	1	5
7	2	14	2	4
8	2	16	3	6
Total	**20**	**110**		**20**

Mean $\bar{x} = \frac{110}{20} = 5.5$

Recalculating deviations from mean = 5.5:
Mean deviation = $\frac{\sum f_i|x_i - \bar{x}|}{\sum f_i} = \frac{22}{20} = 1.1$

Q5.3 [3 marks]

Calculate the standard deviation for the following ungrouped data:
120, 132, 148, 136, 142, 140, 165, 153

Answer: Standard deviation = 13.36

Solution:

x	$x - \bar{x}$	$(x - \bar{x})^2$
120	-19.5	380.25
132	-7.5	56.25
148	8.5	72.25
136	-3.5	12.25
142	2.5	6.25
140	0.5	0.25
165	25.5	650.25
153	13.5	182.25
Total	**0**	**1360**

$n = 8$, $\sum x = 1116$
Mean $\bar{x} = \frac{1116}{8} = 139.5$

Variance = $\frac{\sum(x-\bar{x})^2}{n} = \frac{1360}{8} = 170$

Standard deviation = $\sqrt{170} = 13.04$

Q.5 (B) Attempt any two [8 marks]

Q5.1 [4 marks]

Solve: $\frac{dy}{dx} + \tan x \cdot \tan y = 0$

Answer: $\ln|\cos y| = \ln|\cos x| + C$ or $\cos y = A\cos x$

Solution:
$\frac{dy}{dx} + \tan x \cdot \tan y = 0$
$\frac{dy}{dx} = -\tan x \cdot \tan y$
$\frac{dy}{\tan y} = -\tan x\, dx$
$\cot y\, dy = -\tan x\, dx$

Integrating both sides:
$\int \cot y\, dy = -\int \tan x\, dx$
$\ln|\sin y| = \ln|\cos x| + C_1$
$\ln|\sin y| - \ln|\cos x| = C_1$
$\ln\left|\frac{\sin y}{\cos x}\right| = C_1$

Taking exponential:
$\frac{\sin y}{\cos x} = C$ (where $C = e^{C_1}$)
$\sin y = C\cos x$

Alternative form: $\cos y = A\cos x$ where A is a constant.

Q5.2 [4 marks]

Solve: $\frac{dy}{dx} + 2y = 3e^x$

Answer: $y = e^x + Ce^{-2x}$

Solution:
This is a first-order linear differential equation of the form $\frac{dy}{dx} + Py = Q$
where $P = 2$ and $Q = 3e^x$

Integrating factor: $I.F. = e^{\int P\, dx} = e^{\int 2\, dx} = e^{2x}$

Multiplying the equation by e^{2x}:
$e^{2x}\frac{dy}{dx} + 2e^{2x}y = 3e^{3x}$

The left side is the derivative of ye^{2x}:
$\frac{d}{dx}(ye^{2x}) = 3e^{3x}$

Integrating both sides:
$ye^{2x} = \int 3e^{3x}\,dx = e^{3x} + C$

Therefore: $y = e^x + Ce^{-2x}$

Q5.3 [4 marks]

Solve: $dy + 4xy^2dx = 0$; $y(0) = 1$

Answer: $y = \frac{1}{1+2x^2}$

Solution:
$dy + 4xy^2dx = 0$
$dy = -4xy^2dx$
$\frac{dy}{y^2} = -4x\,dx$

Integrating both sides:
$\int y^{-2}\,dy = \int -4x\,dx$
$-\frac{1}{y} = -2x^2 + C$
$\frac{1}{y} = 2x^2 - C$

Using initial condition $y(0) = 1$:
$\frac{1}{1} = 2(0)^2 - C$
$1 = -C$
$C = -1$

Therefore: $\frac{1}{y} = 2x^2 + 1$
$y = \frac{1}{2x^2+1}$

Formula Cheat Sheet

Matrix Operations

- **Matrix Addition/Subtraction**: Element-wise operation
- **Matrix Multiplication**: $(AB)_{ij} = \sum_k a_{ik}b_{kj}$
- **Transpose**: $(A^T)_{ij} = A_{ji}$
- **Determinant (2×2)**: $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$
- **Inverse (2×2)**: $A^{-1} = \dfrac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
- **Adjoint (2×2)**: $\text{adj} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Differentiation Formulas

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(e^x) = e^x$

- $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- **Chain Rule**: $\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$
- **Product Rule**: $(uv)' = u'v + uv'$
- **Quotient Rule**: $(\frac{u}{v})' = \frac{u'v-uv'}{v^2}$

Integration Formulas

- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$)
- $\int \frac{1}{x} \, dx = \ln |x| + C$
- $\int e^x \, dx = e^x + C$
- $\int \sin x \, dx = -\cos x + C$
- $\int \cos x \, dx = \sin x + C$
- $\int \sec^2 x \, dx = \tan x + C$
- $\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$
- **Integration by Parts**: $\int u \, dv = uv - \int v \, du$

Differential Equations

- **Variable Separable**: $\frac{dy}{dx} = f(x)g(y) \Rightarrow \frac{dy}{g(y)} = f(x)dx$
- **Linear DE**: $\frac{dy}{dx} + Py = Q$, Solution: $y \cdot I.F. = \int Q \cdot I.F. \; dx$
- **Integrating Factor**: $I.F. = e^{\int P \, dx}$

Statistics Formulas

- **Mean**: $\bar{x} = \frac{\sum x_i}{n}$ (ungrouped), $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$ (grouped)
- **Mean Deviation**: $M.D. = \frac{\sum |x_i - \bar{x}|}{n}$ (ungrouped), $M.D. = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$ (grouped)
- **Standard Deviation**: $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$ (ungrouped)
- **Variance**: $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$
- **Range**: Maximum value - Minimum value
- **Combined Mean**: $\bar{x} = \frac{n_1 \bar{x_1} + n_2 \bar{x_2}}{n_1 + n_2}$

Problem-Solving Strategies

Matrix Problems

1. **Check dimensions** before operations
2. **Calculate determinant** first to check if inverse exists
3. **Use cofactor method** for 3×3 matrix inverse
4. **Set up equations** properly for system solving

Differentiation Problems

1. **Identify the type** (implicit, parametric, composite)
2. **Apply appropriate rules** (chain, product, quotient)
3. **Simplify step by step**
4. **Check units** in application problems

Integration Problems

1. **Try standard forms** first
2. **Use substitution** when inner function derivative is present
3. **Apply integration by parts** for products
4. **Check limits** carefully in definite integrals

Differential Equations

1. **Identify the type** (separable, linear, homogeneous)
2. **Apply appropriate method**
3. **Use initial conditions** to find constants
4. **Verify solution** by substitution

Statistics Problems

1. **Organize data** in tabular form
2. **Calculate systematically** using formulas
3. **Use class midpoints** for grouped data
4. **Double-check calculations**

Common Mistakes to Avoid

1. **Matrix multiplication**: Remember it's not commutative ($AB \neq BA$)
2. **Chain rule**: Don't forget to multiply by derivative of inner function
3. **Integration limits**: Be careful with sign changes

4. **Differential equations**: Always include constant of integration
5. **Statistics**: Use correct formulas for grouped vs ungrouped data
6. **Arithmetic errors**: Double-check all calculations
7. **Units**: Maintain proper units throughout calculations

Exam Tips

1. **Read questions carefully -** understand what's being asked
2. **Show all steps -** partial credit is often awarded
3. **Use proper mathematical notation**
4. **Check your answers** when possible
5. **Manage time effectively -** attempt questions you're confident about first
6. **Use formulas correctly -** refer to the formula sheet
7. **For optional questions -** choose the ones you can solve completely
8. **In statistics problems -** organize data clearly before calculations
9. **For differential equations -** verify your solution satisfies the original equation
10. **Practice numerical problems -** accuracy in calculations is crucial