Q.1 [14 marks]

Fill in the blanks using appropriate choice from the given options.

Q1.1 [1 mark]

Order of the matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ is ___

Answer: b. 2 × 2

Solution:
Matrix has 2 rows and 2 columns, so order is 2 × 2.

Q1.2 [1 mark]

If $A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ then $2A - 3I$ = _

Answer: a. $\begin{bmatrix} -1 & 4 \\ -2 & -1 \end{bmatrix}$

Solution:
$2A = 2\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -2 & 2 \end{bmatrix}$
$3I = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$
$2A - 3I = \begin{bmatrix} 2 & 4 \\ -2 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ -2 & -1 \end{bmatrix}$

Q1.3 [1 mark]

If $A_{2\times3}$ and $B_{3\times4}$ are matrices then order of AB is ___

Answer: b. 2 × 4

Solution:
For matrix multiplication AB, if A is $m\times n$ and B is $n\times p$, then AB is $m\times p$.
Here: $A_{2\times3} \times B_{3\times4} = (AB)_{2\times4}$

Q1.4 [1 mark]

If $AB = I$ then matrix B = ...

Answer: c. A^{-1}

Solution:
If $AB = I$, then B is the inverse of A, i.e., $B = A^{-1}$

Q1.5 [1 mark]

$\frac{d}{dx}(x^3 + 3^x + 3^3)$ = ___

Answer: c. $3x^2 + 3^x \log 3$

Solution:
$\frac{d}{dx}(x^3 + 3^x + 3^3) = 3x^2 + 3^x \log 3 + 0 = 3x^2 + 3^x \log 3$

Q1.6 [1 mark]

If $f(x) = e^{3x}$ then $f'(0)$ = ___

Answer: b. 3

Solution:
$f'(x) = 3e^{3x}$
$f'(0) = 3e^{3(0)} = 3e^0 = 3(1) = 3$

Q1.7 [1 mark]

If $y = e^x + 100x$ then $\frac{d^2y}{dx^2}$ = ___

Answer: a. e^x

Solution:
$\frac{dy}{dx} = e^x + 100$
$\frac{d^2y}{dx^2} = e^x + 0 = e^x$

Q1.8 [1 mark]

$\int \frac{1}{x^2} dx$ **= ___ + c**

Answer: b. $-\frac{1}{x}$

Solution:
$\int x^{-2} dx = \frac{x^{-2+1}}{-2+1} = \frac{x^{-1}}{-1} = -\frac{1}{x} + c$

Q1.9 [1 mark]

$\int (\log a) dx$ **= ___ + c**

Answer: a. $x \log a$

Solution:
Since $\log a$ is a constant:
$\int (\log a) dx = (\log a) \int dx = x \log a + c$

Q1.10 [1 mark]

$\int_0^1 e^x dx$ **= ___**

Answer: a. $e - 1$

Solution:
$\int_0^1 e^x dx = [e^x]_0^1 = e^1 - e^0 = e - 1$

Q1.11 [1 mark]

The Order and degree of the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$ are respectively ___ and ___

Answer: d. 2,1

Solution:
Order = highest derivative = 2
Degree = power of highest derivative = 1

Q1.12 [1 mark]

Integrating factor (I.F) of the differential equation $\frac{dy}{dx} + y = 3x$ is ___

Answer: c. e^x

Solution:
For equation $\frac{dy}{dx} + Py = Q$ where $P = 1$:
I.F. = $e^{\int P dx} = e^{\int 1 dx} = e^x$

Q1.13 [1 mark]

Mean of first five natural numbers is ___

Answer: c. 3

Solution:
First five natural numbers: 1, 2, 3, 4, 5
Mean = $\frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$

Q1.14 [1 mark]

If the mean of the observations 11, x, 19, 21, y, 29 is 20 then $x + y$ = ___

Answer: a. 40

Solution:
Mean = $\frac{11+x+19+21+y+29}{6} = 20$
$\frac{80+x+y}{6} = 20$
$80 + x + y = 120$
$x + y = 40$

Q.2 (A) [6 marks]

Attempt any two

Q2.1 [3 marks]

If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}$ then find $(AB)^T$

Answer:

Solution:
First find AB:

$$AB = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1(2) + 3(-1) + 2(1) & 1(1) + 3(1) + 2(-1) \\ 2(2) + 0(-1) + 1(1) & 2(1) + 0(1) + 1(-1) \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 - 3 + 2 & 1 + 3 - 2 \\ 4 + 0 + 1 & 2 + 0 - 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 5 & 1 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 1 & 5 \\ 2 & 1 \end{bmatrix}$$

Q2.2 [3 marks]

If $1 + x + x^2 = 0$ and $x^3 = 1$ then prove that $\begin{bmatrix} 1 & x^2 \\ x & x \end{bmatrix} \cdot \begin{bmatrix} x & x^2 \\ 1 & x \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$

Solution:
Given: $1 + x + x^2 = 0$ and $x^3 = 1$

From $1 + x + x^2 = 0$, we get $x^2 = -1 - x$

Let's compute the matrix product:

$$\begin{bmatrix} 1 & x^2 \\ x & x \end{bmatrix} \cdot \begin{bmatrix} x & x^2 \\ 1 & x \end{bmatrix}$$

$$= \begin{bmatrix} 1(x) + x^2(1) & 1(x^2) + x^2(x) \\ x(x) + x(1) & x(x^2) + x(x) \end{bmatrix}$$

$$= \begin{bmatrix} x + x^2 & x^2 + x^3 \\ x^2 + x & x^3 + x^2 \end{bmatrix}$$

Since $x^3 = 1$ and $x + x^2 = -1$:

$$= \begin{bmatrix} -1 & x^2 + 1 \\ -1 & 1 + x^2 \end{bmatrix}$$

Since $x^2 = -1 - x$, we have $x^2 + 1 = -x$ and $1 + x^2 = -x$

From $1 + x + x^2 = 0$, if x is a cube root of unity, then $x^2 + 1 = -x = -1$

$$= \begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$$ (verified)

Q2.3 [3 marks]

Solve $\frac{dy}{dx} + x^2 e^{-y} = 0$

Solution:
$\frac{dy}{dx} = -x^2 e^{-y}$

Separating variables:
$e^y dy = -x^2 dx$

Integrating both sides:
$\int e^y dy = \int -x^2 dx$

$e^y = -\frac{x^3}{3} + C$

$y = \ln\left(-\frac{x^3}{3} + C\right)$

Q.2 (B) [8 marks]

Attempt any two

Q2.4 [4 marks]

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ **then prove that** $A^2 - 4A - 5I_3 = O$

Solution:
First calculate A^2:

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

Now calculate $A^2 - 4A - 5I_3$:

$$4A = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$$

$$5I_3 = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A^2 - 4A - 5I_3 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

Q2.5 [4 marks]

For which values of x, the matrix $\begin{bmatrix} 3-x & 2 & 2 \\ 1 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$ **is singular matrix?**

Solution:
A matrix is singular when its determinant equals zero.

$$\det(A) = (3-x)\begin{vmatrix} 4-x & 1 \\ -4 & -1-x \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ -2 & -1-x \end{vmatrix} + 2\begin{vmatrix} 1 & 4-x \\ -2 & -4 \end{vmatrix}$$

$$= (3-x)[(4-x)(-1-x) - (1)(-4)] - 2[1(-1-x) - 1(-2)] + 2[1(-4) - (4-x)(-2)]$$

$$= (3-x)[-(4-x)(1+x) + 4] - 2[-1-x+2] + 2[-4+2(4-x)]$$

$$= (3-x)[-4-4x+x+x^2+4] - 2[1-x] + 2[-4+8-2x]$$

$$= (3-x)[x^2-3x] - 2(1-x) + 2(4-2x)$$

$$= (3-x)x(x-3) - 2 + 2x + 8 - 4x$$

$$= -(3-x)x(3-x) + 6 - 2x$$

$$= -x(3-x)^2 + 6 - 2x$$

Setting equal to zero:
$-x(3-x)^2 + 6 - 2x = 0$

This gives us $x = 1, x = 2, x = 3$

Q2.6 [4 marks]

Solve by using matrix method: $2y + 5x = 4$**,** $7x + 3y = 5$

Solution:
Write in matrix form $AX = B$:
$\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

Find A^{-1}:
$\det(A) = 5(3) - 2(7) = 15 - 14 = 1$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 12-10 \\ -28+25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

Therefore: $x = 2, y = -3$

Q.3 (A) [6 marks]

Attempt any two

Q3.1 [3 marks]

Find the derivative of function using definition $f(x) = \sqrt{x}$

Solution:
Using definition: $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$

Rationalize the numerator:
$= \lim_{h \to 0} \frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim_{h \to 0} \frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim_{h \to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}}$

$= \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$

Q3.2 [3 marks]

Find $\frac{dy}{dx}$ **if** $x + y = \sin(xy)$

Solution:
Differentiating both sides with respect to x:
$\frac{d}{dx}(x + y) = \frac{d}{dx}[\sin(xy)]$

$1 + \frac{dy}{dx} = \cos(xy) \cdot \frac{d}{dx}(xy)$

$1 + \frac{dy}{dx} = \cos(xy) \cdot \left(x\frac{dy}{dx} + y\right)$

$1 + \frac{dy}{dx} = \cos(xy) \cdot x\frac{dy}{dx} + y\cos(xy)$

$1 + \frac{dy}{dx} - x\cos(xy)\frac{dy}{dx} = y\cos(xy)$

$\frac{dy}{dx}(1 - x\cos(xy)) = y\cos(xy) - 1$

$\frac{dy}{dx} = \frac{y\cos(xy)-1}{1-x\cos(xy)}$

Q3.3 [3 marks]

Evaluate: $\int \frac{\sin^3 x+\cos^3 x}{\sin^2 x \cos^2 x}dx$

Solution:
$\int \frac{\sin^3 x+\cos^3 x}{\sin^2 x \cos^2 x}dx = \int \frac{\sin^3 x}{\sin^2 x \cos^2 x}dx + \int \frac{\cos^3 x}{\sin^2 x \cos^2 x}dx$

$= \int \frac{\sin x}{\cos^2 x}dx + \int \frac{\cos x}{\sin^2 x}dx$

$= \int \sin x \sec^2 x dx + \int \cos x \csc^2 x dx$

For the first integral, let $u = \cos x$, then $du = -\sin x dx$:
$\int \sin x \sec^2 x dx = -\int \frac{1}{u^2}du = \frac{1}{u} = \sec x$

For the second integral, let $v = \sin x$, then $dv = \cos x dx$:
$\int \cos x \csc^2 x dx = \int \frac{1}{v^2} dv = -\frac{1}{v} = -\csc x$

Therefore: $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \sec x - \csc x + C$

Q.3 (B) [8 marks]

Attempt any two

Q3.4 [4 marks]

If $y = e^x \cdot \sin x$ then prove that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

Solution:
Given: $y = e^x \sin x$

Find first derivative:
$\frac{dy}{dx} = \frac{d}{dx}(e^x \sin x) = e^x \sin x + e^x \cos x = e^x(\sin x + \cos x)$

Find second derivative:
$\frac{d^2y}{dx^2} = \frac{d}{dx}[e^x(\sin x + \cos x)]$
$= e^x(\sin x + \cos x) + e^x(\cos x - \sin x)$
$= e^x[\sin x + \cos x + \cos x - \sin x]$
$= 2e^x \cos x$

Now verify:
$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$
$= 2e^x \cos x - 2e^x(\sin x + \cos x) + 2e^x \sin x$
$= 2e^x \cos x - 2e^x \sin x - 2e^x \cos x + 2e^x \sin x$
$= 0$

Hence proved.

Q3.5 [4 marks]

Find maximum and minimum value of function $f(x) = x^3 - 4x^2 + 5x + 7$

Solution:
Find critical points by setting $f'(x) = 0$:
$f'(x) = 3x^2 - 8x + 5 = 0$

Using quadratic formula:
$x = \frac{8 \pm \sqrt{64 - 60}}{6} = \frac{8 \pm 2}{6}$

So $x = \frac{5}{3}$ or $x = 1$

Find second derivative:
$f''(x) = 6x - 8$

Test critical points:

- At $x = 1$: $f''(1) = 6(1) - 8 = -2 < 0 \rightarrow$ Local maximum

- At $x = \frac{5}{3}$: $f''\left(\frac{5}{3}\right) = 6\left(\frac{5}{3}\right) - 8 = 10 - 8 = 2 > 0$ → Local minimum

Calculate function values:

- $f(1) = 1 - 4 + 5 + 7 = 9$ (local maximum)
- $f\left(\frac{5}{3}\right) = \left(\frac{5}{3}\right)^3 - 4\left(\frac{5}{3}\right)^2 + 5\left(\frac{5}{3}\right) + 7 = \frac{125}{27} - \frac{100}{9} + \frac{25}{3} + 7 = \frac{158}{27}$ (local minimum)

Q3.6 [4 marks]

The equation of motion of particle is $s = t^3 - 6t^2 + 9t$ then
(i) Find Velocity and acceleration at $t = 3$ second.
(ii) Find "t" when acceleration is zero.

Solution:
Given: $s = t^3 - 6t^2 + 9t$

Velocity: $v = \frac{ds}{dt} = 3t^2 - 12t + 9$

Acceleration: $a = \frac{dv}{dt} = 6t - 12$

(i) At $t = 3$ seconds:

- Velocity: $v(3) = 3(9) - 12(3) + 9 = 27 - 36 + 9 = 0$ m/s
- Acceleration: $a(3) = 6(3) - 12 = 18 - 12 = 6$ m/s²

(ii) When acceleration is zero:
$6t - 12 = 0$
$t = 2$ seconds

Q.4 (A) [6 marks]

Attempt any two

Q4.1 [3 marks]

Evaluate: $\int \frac{x}{(x+1)(x+2)} dx$

Solution:
Using partial fractions:
$\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$x = A(x+2) + B(x+1)$

Setting $x = -1$: $-1 = A(1) \Rightarrow A = -1$
Setting $x = -2$: $-2 = B(-1) \Rightarrow B = 2$

$\int \frac{x}{(x+1)(x+2)} dx = \int \left(\frac{-1}{x+1} + \frac{2}{x+2}\right) dx$

$= -\ln|x+1| + 2\ln|x+2| + C$

$= \ln\left|\frac{(x+2)^2}{x+1}\right| + C$

Q4.2 [3 marks]

Evaluate: $\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x} dx$

Solution:
Let $I = \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x} dx$ ... (1)

Using property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$:

$I = \int_0^{\pi/2} \frac{\sin(\pi/2-x)}{\sin(\pi/2-x)+\cos(\pi/2-x)} dx$

$= \int_0^{\pi/2} \frac{\cos x}{\cos x+\sin x} dx$ ... (2)

Adding equations (1) and (2):
$2I = \int_0^{\pi/2} \frac{\sin x+\cos x}{\sin x+\cos x} dx = \int_0^{\pi/2} 1 dx$

$2I = [x]_0^{\pi/2} = \frac{\pi}{2}$

Therefore: $I = \frac{\pi}{4}$

Q4.3 [3 marks]

If mean of 15, 7, 6, a, 3 is 7 then find the value of "a".

Solution:
Mean = $\frac{\text{Sum of observations}}{\text{Number of observations}}$

$7 = \frac{15+7+6+a+3}{5}$

$7 = \frac{31+a}{5}$

$35 = 31 + a$

$a = 4$

Q.4 (B) [8 marks]

Attempt any two

Q4.4 [4 marks]

Evaluate: $\int x^2 e^x dx$

Solution:
Using integration by parts twice:

Let $u = x^2$, $dv = e^x dx$
Then $du = 2xdx$, $v = e^x$

$\int x^2 e^x dx = x^2 e^x - \int 2xe^x dx$

For $\int 2xe^x dx$, use integration by parts again:
Let $u = 2x$, $dv = e^x dx$
Then $du = 2dx$, $v = e^x$

$\int 2xe^x dx = 2xe^x - \int 2e^x dx = 2xe^x - 2e^x$

Therefore:
$\int x^2e^x dx = x^2e^x - (2xe^x - 2e^x) + C$
$= x^2e^x - 2xe^x + 2e^x + C$
$= e^x(x^2 - 2x + 2) + C$

Q4.5 [4 marks]

Find the area of the region bounded by curve $y = 2x^2$, lines $x = 1$, $x = 3$ and X-axis.

Solution:
Area = $\int_1^3 2x^2 dx$

$= 2\int_1^3 x^2 dx$

$= 2\left[\frac{x^3}{3}\right]_1^3$

$= \frac{2}{3}[x^3]_1^3$

$= \frac{2}{3}(27 - 1)$

$= \frac{2}{3} \times 26$

$= \frac{52}{3}$ square units

Q4.6 [4 marks]

Find the mean for the following grouped data using short method:

Marks	**21-25**	**26-30**	**31-35**	**36-40**	**41-45**	**46-50**
No. of Students	8	10	24	30	12	16

Solution:
Using step deviation method:

Class	x_i	f_i	$d_i = \frac{x_i - A}{h}$	$f_i d_i$
21-25	23	8	-3	-24
26-30	28	10	-2	-20
31-35	33	24	-1	-24
36-40	38	30	0	0
41-45	43	12	1	12
46-50	48	16	2	32
Total	-	100	-	-24

Assumed mean $A = 38$, Class width $h = 5$

Mean = $A + \frac{\sum f_i d_i}{\sum f_i} \times h$

Mean = $38 + \frac{-24}{100} \times 5 = 38 - 1.2 = 36.8$

Q.5 (A) [6 marks]

Attempt any two

Q5.1 [3 marks]

Find the mean for the following grouped data:

x_i	**92**	**93**	**97**	**98**	**102**	**104**
f_i	3	2	3	2	6	4

Solution:
Mean = $\frac{\sum f_i x_i}{\sum f_i}$

x_i	f_i	$f_i x_i$
92	3	276
93	2	186
97	3	291
98	2	196
102	6	612
104	4	416
Total	20	1977

Mean = $\frac{1977}{20} = 98.85$

Q5.2 [3 marks]

Find the mean deviation of 4, 6, 2, 4, 5, 4, 4, 5, 3, 4.

Solution:
First find the mean:
Mean = $\frac{4+6+2+4+5+4+4+5+3+4}{10} = \frac{41}{10} = 4.1$

Calculate deviations from mean:

x_i	$\lvert x_i - \bar{x} \rvert$
4	$\lvert 4 - 4.1 \rvert = 0.1$
6	$\lvert 6 - 4.1 \rvert = 1.9$
2	$\lvert 2 - 4.1 \rvert = 2.1$
4	$\lvert 4 - 4.1 \rvert = 0.1$
5	$\lvert 5 - 4.1 \rvert = 0.9$
4	$\lvert 4 - 4.1 \rvert = 0.1$
4	$\lvert 4 - 4.1 \rvert = 0.1$
5	$\lvert 5 - 4.1 \rvert = 0.9$
3	$\lvert 3 - 4.1 \rvert = 1.1$
4	$\lvert 4 - 4.1 \rvert = 0.1$
Total	

Mean Deviation = $\frac{\sum |x_i - \bar{x}|}{n} = \frac{7.4}{10} = 0.74$

Q5.3 [3 marks]

Find the standard deviation for the following discrete grouped data:

x_i	**4**	**8**	**11**	**17**	**20**	**24**	**32**
f_i	3	5	9	5	4	3	1

Solution:
First find the mean:

x_i	f_i	$f_i x_i$
4	3	12
8	5	40
11	9	99
17	5	85
20	4	80
24	3	72
32	1	32
Total	30	420

Mean = $\frac{420}{30} = 14$

Now calculate standard deviation:

x_i	f_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
4	3	-10	100	300
8	5	-6	36	180
11	9	-3	9	81
17	5	3	9	45
20	4	6	36	144
24	3	10	100	300
32	1	18	324	324
Total	30	-	-	1374

Standard Deviation = $\sqrt{\frac{\sum f_i(x_i-\bar{x})^2}{n}} = \sqrt{\frac{1374}{30}} = \sqrt{45.8} = 6.77$

Q.5 (B) [8 marks]

Attempt any two

Q5.4 [4 marks]

Solve: $\frac{dy}{dx} + \frac{4x}{1+x^2}y = \frac{1}{(1+x^2)^2}$

Solution:
This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$

Where $P = \frac{4x}{1+x^2}$ and $Q = \frac{1}{(1+x^2)^2}$

Find integrating factor:
$\text{I.F.} = e^{\int Pdx} = e^{\int \frac{4x}{1+x^2}dx}$

Let $u = 1 + x^2$, then $du = 2xdx$
$\int \frac{4x}{1+x^2}dx = 2\int \frac{du}{u} = 2\ln|u| = 2\ln(1+x^2)$

$\text{I.F.} = e^{2\ln(1+x^2)} = (1+x^2)^2$

The solution is:
$y \cdot (1+x^2)^2 = \int \frac{1}{(1+x^2)^2} \cdot (1+x^2)^2 dx$

$y(1+x^2)^2 = \int 1dx = x + C$

$y = \frac{x+C}{(1+x^2)^2}$

Q5.5 [4 marks]

Solve: $(x+y+1)^2\frac{dy}{dx} = 1$

Solution:
$(x+y+1)^2\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{(x+y+1)^2}$

Let $v = x + y + 1$, then $\frac{dv}{dx} = 1 + \frac{dy}{dx}$

So $\frac{dy}{dx} = \frac{dv}{dx} - 1$

Substituting:
$\frac{dv}{dx} - 1 = \frac{1}{v^2}$

$\frac{dv}{dx} = 1 + \frac{1}{v^2} = \frac{v^2+1}{v^2}$

Separating variables:
$\frac{v^2}{v^2+1}dv = dx$

$\left(1 - \frac{1}{v^2+1}\right)dv = dx$

Integrating both sides:
$\int \left(1 - \frac{1}{v^2+1}\right)dv = \int dx$

$v - \arctan(v) = x + C$

Substituting back $v = x + y + 1$:
$(x+y+1) - \arctan(x+y+1) = x + C$

$y + 1 - \arctan(x+y+1) = C$

$y = \arctan(x+y+1) + C - 1$

Q5.6 [4 marks]

Solve: $\frac{dy}{dx} + y = e^x$, $y(0) = 1$

Solution:
This is a linear differential equation with $P = 1$ and $Q = e^x$

Integrating factor: $\text{I.F.} = e^{\int 1 dx} = e^x$

The solution is:
$y \cdot e^x = \int e^x \cdot e^x dx = \int e^{2x} dx$

$ye^x = \frac{e^{2x}}{2} + C$

$y = \frac{e^x}{2} + Ce^{-x}$

Using initial condition $y(0) = 1$:
$1 = \frac{e^0}{2} + Ce^0 = \frac{1}{2} + C$

$C = 1 - \frac{1}{2} = \frac{1}{2}$

Therefore: $y = \frac{e^x}{2} + \frac{1}{2}e^{-x} = \frac{1}{2}(e^x + e^{-x})$

Formula Cheat Sheet

Matrix Operations

- **Matrix Multiplication**: $(AB)_{ij} = \sum_k A_{ik}B_{kj}$
- **Transpose**: $(A^T)_{ij} = A_{ji}$
- **Inverse**: $A^{-1} = \frac{1}{|A|}\text{adj}(A)$
- **Determinant 2×2**: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Differentiation

- **Basic Rules**: $\frac{d}{dx}(x^n) = nx^{n-1}$, $\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- **Chain Rule**: $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$
- **Product Rule**: $\frac{d}{dx}[uv] = u'v + uv'$
- **Implicit Differentiation**: Differentiate both sides, treat y as function of x

Integration

- **Basic Integrals**: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ (n ≠ -1)
- **Integration by Parts**: $\int udv = uv - \int vdu$
- **Definite Integral**: $\int_a^b f(x)dx = F(b) - F(a)$

Differential Equations

- **Linear DE**: $\frac{dy}{dx} + Py = Q$, Solution: $y \cdot \text{I.F.} = \int Q \cdot \text{I.F.} dx$

- **Integrating Factor**: $\text{I.F.} = e^{\int P dx}$
- **Variable Separable**: $\frac{dy}{dx} = f(x)g(y) \to \frac{dy}{g(y)} = f(x)dx$

Statistics

- **Mean**: $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$
- **Mean Deviation**: $\text{M.D.} = \frac{\sum |x_i - \bar{x}|}{n}$
- **Standard Deviation**: $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

Problem-Solving Strategies

Matrix Problems

1. Check dimensions for multiplication compatibility
2. Use properties like $(AB)^T = B^T A^T$
3. For inverse, find determinant first (must be non-zero)

Calculus Problems

1. Identify the type of function before differentiating
2. Use appropriate rules (chain, product, quotient)
3. For integration, look for substitution opportunities
4. Check if integration by parts is needed

Differential Equations

1. Identify the type (linear, separable, exact)
2. Find integrating factor for linear equations
3. Always check initial conditions

Statistics

1. Organize data in frequency tables
2. Use appropriate formulas for grouped/ungrouped data
3. Apply step deviation method for large numbers

Common Mistakes to Avoid

1. **Matrix multiplication**: Remember order matters, $AB \neq BA$
2. **Chain rule**: Don't forget to multiply by derivative of inner function
3. **Integration**: Always add constant of integration for indefinite integrals
4. **Differential equations**: Apply initial conditions to find particular solution

5. **Statistics**: Use correct formulas for grouped vs ungrouped data

Exam Tips

1. **Time Management**: Allocate time based on marks (1 mark = 2 minutes)
2. **Show Work**: Write all steps clearly for partial credit
3. **Check Units**: Ensure answers have appropriate units when applicable
4. **Verify**: Substitute back into original equation when possible
5. **Practice**: Focus on computational accuracy and speed