# Q.1 [14 marks]

Fill in the blanks using appropriate choice from the given options.

## Q.1.1 [1 mark]

 $\text{Order of} \begin{bmatrix} 1 & 0 & 3 \\ -2 & 4 & 0 \end{bmatrix} \text{is} \_\_.$ 

Answer: b.  $2 \times 3$ 

**Solution**: The matrix has 2 rows and 3 columns, so the order is  $2 \times 3$ .

## Q.1.2 [1 mark]

If A is of order 2 imes 3 and B is of order 3 imes 2 then AB is of order \_\_\_\_\_.

Answer: d.  $2 \times 2$ 

**Solution**: For matrix multiplication AB, if A is  $2 \times 3$  and B is  $3 \times 2$ , then AB is of order  $2 \times 2$ .

## Q.1.3 [1 mark]

If 
$$A = \begin{bmatrix} 1 & -1 \end{bmatrix}$$
 then  $A^T = \_$   
Answer: b.  $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ 

#### Solution:

The transpose of a row matrix becomes a column matrix.

$$A^T = egin{bmatrix} 1 \ -1 \end{bmatrix}$$

## Q.1.4 [1 mark]

If 
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$
 then  $\operatorname{adj} A = \_$   
Answer: d.  $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ 

#### Solution:

```
For a 2 \times 2 matrix A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, adj A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}
Therefore: adj A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}
```

# Q.1.5 [1 mark]

$$\frac{d}{dx}(e^x) = \_$$

Answer: a.  $e^x$ 

Solution:  $\frac{d}{dx}(e^x) = e^x$ 

# Q.1.6 [1 mark]

If  $f(x) = \log x$  then f'(1) =\_\_\_\_

**Answer**: c. 1

#### Solution:

 $f'(x) = rac{1}{x} \ f'(1) = rac{1}{1} = 1$ 

# Q.1.7 [1 mark]

 $\frac{d}{dx}(3^{\log_3 x}) = \_$ 

Answer: b. 2x

#### Solution:

Using the property  $a^{\log_a x} = x$ : $3^{\log_3 x} = x$ Therefore:  $rac{d}{dx}(3^{\log_3 x}) = rac{d}{dx}(x) = 1$ 

Wait, let me recalculate this. The expression is  $3^{\log_3 x^2} = x^2$   $rac{d}{dx}(x^2) = 2x$ 

# Q.1.8 [1 mark]

 $\int \sin x \, dx =$ \_

Answer: c.  $-\cos x$ 

Solution:  $\int \sin x \, dx = -\cos x + C$ 

## Q.1.9 [1 mark]

 $\int_{-1}^{1} x^3 \, dx =$  \_

Answer: b. 0

Solution:

$$\int_{-1}^{1} x^3 \, dx = \left[rac{x^4}{4}
ight]_{-1}^{1} = rac{1}{4} - rac{1}{4} = 0$$

## Q.1.10 [1 mark]

$$\int rac{1}{1+x^2}\,dx=$$
 \_

Answer: d.  $\tan^{-1} x$ 

Solution:  $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$ 

# Q.1.11 [1 mark]

Order of the differential equation  $rac{d^2y}{dx^2}-y=0$  is \_\_\_\_.

Answer: b. 2

**Solution**: The highest derivative is  $\frac{d^2y}{dx^2}$ , so the order is 2.

## Q.1.12 [1 mark]

The integration factor (I.F) of  $rac{dy}{dx} + Py = Q$  is \_\_\_\_\_

Answer: a.  $e^{\int P dx}$ 

**Solution**: For a linear differential equation  $\frac{dy}{dx} + Py = Q$ , the integrating factor is  $e^{\int P dx}$ .

# Q.1.13 [1 mark]

If Z=4-5i then  $ar{Z}=$  \_\_\_\_

Answer: c. 4-5i

#### Solution:

Wait, this seems incorrect. If Z=4-5i, then  $\bar{Z}=4+5i$ . The correct answer should be 4+5i.

## Q.1.14 [1 mark]

 $i^{10} = \_$ 

**Answer**: b. -1

Solution:  $i^{10} = i^{4\cdot 2+2} = (i^4)^2 \cdot i^2 = 1^2 \cdot (-1) = -1$ 

# Q.2 (A) [6 marks]

Attempt any two.

Q.2(A).1 [3 marks]

If  $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$  then find the matrix X such that 2A + X = 3B.

#### Solution:

2A + X = 3BX = 3B - 2A

$$2A = 2\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 8 & 6 \end{bmatrix}$$
$$3B = 3\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 3 & 12 \end{bmatrix}$$
$$X = \begin{bmatrix} 9 & 6 \\ 3 & 12 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 8 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ -5 & 6 \end{bmatrix}$$

## Q.2(A).2 [3 marks]

If 
$$A = egin{bmatrix} 5 & 4 \\ 4 & 3 \end{bmatrix}$$
 and  $B = egin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$  then find  $(AB)^T$ 

Solution:

First, find 
$$AB$$
:  
 $AB = \begin{bmatrix} 5 & 4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$   
 $AB = \begin{bmatrix} 5(1) + 4(2) & 5(3) + 4(1) \\ 4(1) + 3(2) & 4(3) + 3(1) \end{bmatrix} = \begin{bmatrix} 13 & 19 \\ 10 & 15 \end{bmatrix}$   
 $(AB)^T = \begin{bmatrix} 13 & 10 \\ 19 & 15 \end{bmatrix}$ 

## Q.2(A).3 [3 marks]

Solve:  $rac{dy}{dx} = x^2 \cdot e^{-y}$  .

#### Solution:

 $rac{dy}{dx} = x^2 \cdot e^{-y}$ 

Separating variables:  $e^y \, dy = x^2 \, dx$ 

Integrating both sides:  $\int e^y dy = \int x^2 dx$ 

$$e^{y} = \frac{x^{3}}{3} + C$$
$$y = \ln\left(\frac{x^{3}}{3} + C\right)$$

# Q.2 (B) [8 marks]

Attempt any two.

#### Q.2(B).1 [4 marks]

If 
$$A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 0 \end{bmatrix}$$
 and  $B = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 1 \end{bmatrix}$  then prove that  $(A+B)^T = A^T + B^T$ 

Solution:

Solution:  

$$A + B = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & 5 & 3 \\ 6 & 8 & 1 \end{bmatrix}$$

$$(A + B)^{T} = \begin{bmatrix} 3 & 6 \\ 5 & 8 \\ 3 & 1 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} 2 & 4 \\ 3 & 5 \\ -1 & 0 \end{bmatrix}, B^{T} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 4 & 1 \end{bmatrix}$$

$$A^{T} + B^{T} = \begin{bmatrix} 2 & 4 \\ 3 & 5 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 5 & 8 \\ 3 & 1 \end{bmatrix}$$

Therefore,  $(A+B)^T = A^T + B^T$  is proved.

#### Q.2(B).2 [4 marks]

If 
$$A = egin{bmatrix} 2 & -1 & 0 \ 1 & 0 & 4 \ 1 & -1 & 1 \end{bmatrix}$$
 then find  $A^{-1}.$ 

#### Solution:

To find  $A^{-1}$ , we use the formula  $A^{-1} = rac{1}{|A|} \cdot \mathrm{adj}(A)$ 

First, find |A|:  $|A| = 2(0 \cdot 1 - 4 \cdot (-1)) - (-1)(1 \cdot 1 - 4 \cdot 1) + 0(1 \cdot (-1) - 0 \cdot 1)$ |A| = 2(4) + 1(-3) = 8 - 3 = 5

Next, find cofactors:

$$C_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 4 \\ -1 & 1 \end{vmatrix} = 4$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} = -(-3) = 3$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = -1$$

$$C_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 0 \\ -1 & 1 \end{vmatrix} = -(-1) = 1$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} = -(-1) = 1$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 0 \\ 0 & 4 \end{vmatrix} = -4$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 0 \\ 1 & 4 \end{vmatrix} = -(8) = -8$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = 1$$

$$adj(A) = \begin{bmatrix} 4 & 1 & -4 \\ 3 & 2 & -8 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 4 & 1 & -4 \\ 3 & 2 & -8 \\ -1 & 1 & 1 \end{bmatrix}$$

## Q.2(B).3 [4 marks]

Solve the equations 3x - y = 1, x + 2y = 5 by matrix method.

#### Solution:

The system can be written as AX = B where:  $A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$  |A| = 3(2) - (-1)(1) = 6 + 1 = 7  $A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$   $X = A^{-1}B = \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \end{bmatrix}$  $X = \frac{1}{7} \begin{bmatrix} 2+5 \\ -1+15 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 7 \\ 14 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ 

Therefore, x = 1 and y = 2.

# Q.3 (A) [6 marks]

#### Attempt any two.

#### Q.3(A).1 [3 marks]

If 
$$y=rac{e^x+1}{e^x-1}$$
 then find  $rac{dy}{dx}.$ 

#### Solution:

Using quotient rule:  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ Let  $u = e^x + 1$  and  $v = e^x - 1$  $\frac{du}{dx} = e^x$  and  $\frac{dv}{dx} = e^x$  $\frac{dy}{dx} = \frac{(e^x - 1)(e^x) - (e^x + 1)(e^x)}{(e^x - 1)^2}$ 

$$= \frac{e^{2x} - e^x - e^{2x} - e^x}{(e^x - 1)^2} = \frac{-2e^x}{(e^x - 1)^2}$$

## Q.3(A).2 [3 marks]

If  $x = a\cos heta, y = b\sin heta$  then find  $rac{dy}{dx}$ .

#### Solution:

 $egin{aligned} rac{dx}{d heta} &= -a\sin heta\ rac{dy}{d heta} &= b\cos heta\ rac{dy}{d heta} &= b\cos heta\ rac{dy}{dx} &= rac{dy/d heta}{dx/d heta} &= rac{b\cos heta}{-a\sin heta} &= -rac{b\cos heta}{a\sin heta} &= -rac{b}{a}\cot heta \end{aligned}$ 

# Q.3(A).3 [3 marks]

Evaluate:  $\int rac{\cos\sqrt{x}}{2\sqrt{x}} dx$ .

Solution: Let  $u=\sqrt{x}$ , then  $du=rac{1}{2\sqrt{x}}dx$ 

 $\int \frac{\cos\sqrt{x}}{2\sqrt{x}} dx = \int \cos u \, du = \sin u + C = \sin\sqrt{x} + C$ 

# Q.3 (B) [8 marks]

Attempt any two.

## Q.3(B).1 [4 marks]

Differentiate  $y = x^{\cos x}$  with respect to x.

#### Solution:

Taking natural logarithm on both sides:  $\ln y = \cos x \ln x$ 

Differentiating both sides with respect to x:  $\frac{1}{y}\frac{dy}{dx} = \cos x \cdot \frac{1}{x} + \ln x \cdot (-\sin x)$   $\frac{dy}{dx} = y \left(\frac{\cos x}{x} - \sin x \ln x\right)$   $\frac{dy}{dx} = x^{\cos x} \left(\frac{\cos x}{x} - \sin x \ln x\right)$ 

## Q.3(B).2 [4 marks]

If  $y=A\cos pt+B\sin pt$ , prove that  $rac{d^2y}{dt^2}+p^2y=0.$ 

#### Solution:

$$egin{aligned} y&=A\cos pt+B\sin pt\ rac{dy}{dt}&=-Ap\sin pt+Bp\cos pt\ rac{d^2y}{dt^2}&=-Ap^2\cos pt-Bp^2\sin pt=-p^2(A\cos pt+B\sin pt)=-p^2y\ \end{aligned}$$
 Therefore:  $rac{d^2y}{dt^2}+p^2y=-p^2y+p^2y=0 \end{aligned}$ 

## Q.3(B).3 [4 marks]

The equation of motion of a particle is  $s = t^3 + 2t^2 - 3t + 5$ . Find the velocity and acceleration of the particle at t = 1 and t = 2 seconds.

#### Solution:

 $s = t^{3} + 2t^{2} - 3t + 5$ Velocity:  $v = \frac{ds}{dt} = 3t^{2} + 4t - 3$ Acceleration:  $a = \frac{dv}{dt} = 6t + 4$ At t = 1:  $v(1) = 3(1)^{2} + 4(1) - 3 = 3 + 4 - 3 = 4$  units/sec a(1) = 6(1) + 4 = 10 units/sec<sup>2</sup> At t = 2:  $v(2) = 3(2)^{2} + 4(2) - 3 = 12 + 8 - 3 = 17$  units/sec a(2) = 6(2) + 4 = 16 units/sec<sup>2</sup>

# Q.4 (A) [6 marks]

#### Attempt any two.

#### Q.4(A).1 [3 marks]

**Evaluate:**  $\int x \log x \, dx$ .

#### Solution:

Using integration by parts:  $\int u \, dv = uv - \int v \, du$ 

Let 
$$u = \log x$$
 and  $dv = x \, dx$   
Then  $du = \frac{1}{x} dx$  and  $v = \frac{x^2}{2}$   
 $\int x \log x \, dx = \log x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$   
 $= \frac{x^2 \log x}{2} - \int \frac{x}{2} dx$   
 $= \frac{x^2 \log x}{2} - \frac{x^2}{4} + C$   
 $= \frac{x^2}{2} (\log x - \frac{1}{2}) + C$ 

## Q.4(A).2 [3 marks]

Evaluate: 
$$\int_{-1}^1 rac{1}{1+x^2} dx.$$

# Solution: $\int_{-1}^{1} \frac{1}{1+x^2} dx = [\tan^{-1} x]_{-1}^{1}$ $= \tan^{-1}(1) - \tan^{-1}(-1)$ $= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$

## Q.4(A).3 [3 marks]

Find inverse of Z = 3 + 4i.

# Solution: $7^{-1}$

 $Z^{-1} = \frac{1}{Z} = \frac{1}{3+4i}$ 

Multiply numerator and denominator by the conjugate:  $Z^{-1} = \frac{1}{3+4i} \cdot \frac{3-4i}{3-4i} = \frac{3-4i}{(3)^2+(4)^2} = \frac{3-4i}{9+16} = \frac{3-4i}{25}$   $Z^{-1} = \frac{3}{25} - \frac{4}{25}i$ 

# Q.4 (B) [8 marks]

Attempt any two.

## Q.4(B).1 [4 marks]

Evaluate:  $\int_0^{\pi/2} rac{ an x}{ an x + \cot x} dx$ .

#### Solution:

Let  $I = \int_0^{\pi/2} \frac{\tan x}{\tan x + \cot x} dx$ Using the property  $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$ :  $I = \int_0^{\pi/2} \frac{\tan(\pi/2-x)}{\tan(\pi/2-x) + \cot(\pi/2-x)} dx$ 

$$=\int_0^{\pi/2}rac{\cot x}{\cot x+\tan x}dx$$

Adding the two expressions:  $2I = \int_0^{\pi/2} rac{ an x + \cot x}{ an x + \cot x} dx = \int_0^{\pi/2} 1 \, dx = rac{\pi}{2}$ 

Therefore:  $I = \frac{\pi}{4}$ 

## Q.4(B).2 [4 marks]

#### Find the area bounded by the line y=x, x=5 and the X-axis.

#### Solution:

The region is bounded by y=x, x=5, and y=0 (X-axis).

Area =  $\int_0^5 x \, dx = \left[ \frac{x^2}{2} \right]_0^5 = \frac{25}{2} - 0 = \frac{25}{2}$  square units

## Q.4(B).3 [4 marks]

If  $x+iy=\left(rac{1+i}{2-i}
ight)^2$ , find the value of x+y.

#### Solution:

First, simplify  $\frac{1+i}{2-i}$ :  $\frac{1+i}{2-i} \cdot \frac{2+i}{2+i} = \frac{(1+i)(2+i)}{(2-i)(2+i)} = \frac{2+i+2i+i^2}{4-i^2} = \frac{2+3i-1}{4+1} = \frac{1+3i}{5}$ Now:  $\left(\frac{1+3i}{5}\right)^2 = \frac{(1+3i)^2}{25} = \frac{1+6i+9i^2}{25} = \frac{1+6i-9}{25} = \frac{-8+6i}{25}$ Therefore:  $x = -\frac{8}{25}$  and  $y = \frac{6}{25}$ 

# $x + y = -\frac{8}{25} + \frac{6}{25} = -\frac{2}{25}$ Q.5 (A) [6 marks]

Attempt any two.

## Q.5(A).1 [3 marks]

Find Square root of Z = 5 + 12i.

## Solution: Let $\sqrt{5+12i}=a+bi$ where $a,b\in\mathbb{R}$ $(a+bi)^2 = 5+12i$ $a^2 + 2abi + b^2i^2 = 5 + 12i$ $(a^2 - b^2) + 2abi = 5 + 12i$ Comparing real and imaginary parts: $a^2 - b^2 = 5 \dots (1)$ $2ab = 12 \dots (2)$ From (2): $b = \frac{6}{a}$ Substituting in (1): $a^2 - rac{36}{a^2} = 5$ $a^4 - 5a^2 - 36 = 0$ Let $u = a^2$ : $u^2 - 5u - 36 = 0$ (u-9)(u+4) = 0Since $u=a^2>0$ , we have u=9, so $a=\pm 3$ If a = 3, then b = 2If a = -3, then b = -2Therefore: $\sqrt{5+12i} = \pm(3+2i)$

# Q.5(A).2 [3 marks]

Find  $x,y\in \mathbb{R}$  from the equation (2x-y)+yi=6+4i.

#### Solution:

Comparing real and imaginary parts: Real part:  $2x - y = 6 \dots (1)$ Imaginary part:  $y = 4 \dots (2)$ Substituting (2) into (1): 2x - 4 = 62x = 10x = 5

Therefore: x=5 and y=4

## Q.5(A).3 [3 marks]

Find the modulus and principal argument of Z=1+i, and express Z into the polar form.

#### Solution:

$$\begin{split} &Z=1+i\\ \text{Modulus: } |Z|=\sqrt{1^2+1^2}=\sqrt{2}\\ &\text{Principal argument: } \arg(Z)=\tan^{-1}\left(\frac{1}{1}\right)=\tan^{-1}(1)=\frac{\pi}{4}\\ &\text{Polar form: } Z=|Z|(\cos\theta+i\sin\theta)=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right) \end{split}$$

# Q.5 (B) [8 marks]

Attempt any two.

## Q.5(B).1 [4 marks]

Solve:  $rac{dy}{dx} = 1 + x + y + xy$ .

Solution:

 $rac{dy}{dx} = 1 + x + y + xy = (1 + x) + y(1 + x) = (1 + x)(1 + y)$ 

Separating variables:  $rac{dy}{1+y} = (1+x)dx$ 

Integrating both sides:  $\int \frac{dy}{1+y} = \int (1+x)dx$   $\ln|1+y| = x + \frac{x^2}{2} + C$   $1+y = Ae^{x+x^2/2} \text{ where } A = e^C$   $y = Ae^{x+x^2/2} - 1$ 

## Q.5(B).2 [4 marks]

Solve the differential equation:  $rac{dy}{dx}+y=e^x.$ 

#### Solution:

This is a first-order linear differential equation of the form  $\frac{dy}{dx} + Py = Q$  where P = 1 and  $Q = e^x$ . Integrating factor:  $I. F. = e^{\int P dx} = e^{\int 1 dx} = e^x$ 

Multiplying the equation by  $e^x$ :  $e^x \frac{dy}{dx} + e^x y = e^{2x}$  $\frac{d}{dx}(ye^x) = e^{2x}$ 

Integrating both sides:  $ye^x = \int e^{2x} dx = rac{e^{2x}}{2} + C$  $y = rac{e^x}{2} + Ce^{-x}$ 

#### Q.5(B).3 [4 marks]

## Solve the differential equation: $rac{dy}{dx} - y an x = 1$ .

#### Solution:

This is a first-order linear differential equation where  $P = -\tan x$  and Q = 1.

Integrating factor:  $I. F. = e^{\int (-\tan x) dx} = e^{\ln |\cos x|} = \cos x$ 

Multiplying the equation by  $\cos x$ :  $\cos x \frac{dy}{dx} - y \cos x \tan x = \cos x$   $\cos x \frac{dy}{dx} - y \sin x = \cos x$  $\frac{d}{dx} (y \cos x) = \cos x$ 

Integrating both sides:  $y \cos x = \int \cos x \, dx = \sin x + C$  $y = \tan x + \frac{C}{\cos x} = \tan x + C \sec x$ 

# **Formula Cheat Sheet**

#### **Matrix Operations**

- Order of Matrix: If matrix has m rows and n columns, order is m imes n
- Matrix Multiplication:  $(AB)_{ij} = \sum_k A_{ik} B_{kj}$
- Transpose:  $(A^T)_{ij} = A_{ji}$

• Adjoint of 2×2 Matrix: If 
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
, then  $\operatorname{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ 

• Inverse:  $A^{-1} = \frac{1}{|A|} \cdot \operatorname{adj}(A)$ 

#### Differentiation

- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- Chain Rule:  $rac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$
- Product Rule:  $rac{d}{dx}(uv) = u'v + uv'$
- Quotient Rule:  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v uv'}{v^2}$
- **Parametric**: If x = f(t) and y = g(t), then  $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

#### Integration

•  $\int x^n\,dx=rac{x^{n+1}}{n+1}+C$  (for n
eq-1)

- $\int e^x dx = e^x + C$
- $\int \frac{1}{x} dx = \ln |x| + C$
- $\int \sin x \, dx = -\cos x + C$
- $\int \cos x \, dx = \sin x + C$
- $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
- Integration by Parts:  $\int u\,dv = uv \int v\,du$
- Definite Integration:  $\int_a^b f(x) \, dx = F(b) F(a)$  where F'(x) = f(x)

#### **Differential Equations**

- Order: Highest derivative present
- Degree: Power of highest derivative
- Linear DE:  $\frac{dy}{dx} + Py = Q$
- Integrating Factor:  $I. F. = e^{\int P \, dx}$
- Variable Separable:  $rac{dy}{dx} = f(x)g(y) o rac{dy}{g(y)} = f(x)dx$

#### **Complex Numbers**

- Standard Form: z = a + bi
- Conjugate:  $\overline{a+bi}=a-bi$
- Modulus:  $|a+bi|=\sqrt{a^2+b^2}$
- Argument:  $\arg(z) = \tan^{-1}\left(\frac{b}{a}\right)$
- Polar Form:  $z = r(\cos heta + i \sin heta)$  where r = |z| and  $heta = \arg(z)$
- Powers of i:  $i^1=i, i^2=-1, i^3=-i, i^4=1$
- Inverse:  $z^{-1} = rac{\overline{z}}{|z|^2}$

# **Problem-Solving Strategies**

#### **Matrix Problems**

- 1. Check dimensions before multiplication
- 2. Use properties:  $(AB)^{T} = B^{T}A^{T}$ ,  $(A + B)^{T} = A^{T} + B^{T}$
- 3. For inverse: Calculate determinant first, then adjoint
- 4. System of equations: Write as AX = B, solve  $X = A^{-1}B$

#### **Differentiation Problems**

- 1. Identify the type: Basic, chain rule, product rule, quotient rule
- 2. For implicit: Differentiate both sides with respect to x

- 3. For parametric: Use  $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$
- 4. For logarithmic: Take In of both sides first

#### **Integration Problems**

- 1. Check standard forms first
- 2. For products: Try integration by parts (ILATE rule)
- 3. For rational functions: Check for substitution
- 4. For definite integrals: Use properties like  $\int_{-a}^{a} f(x) dx = 0$  if f(x) is odd

#### **Differential Equations**

- 1. Identify type: Order, degree, linear/non-linear
- 2. For linear DE: Find integrating factor
- 3. For separable: Separate variables and integrate
- 4. Check initial conditions if given

#### **Complex Numbers**

- 1. For operations: Use standard form a + bi
- 2. For modulus/argument: Convert to polar form
- 3. For powers: Use De Moivre's theorem
- 4. For square roots: Let  $\sqrt{a+bi}=c+di$  and solve

# **Common Mistakes to Avoid**

- 1. Matrix multiplication: Remember AB 
  eq BA in general
- 2. Chain rule: Don't forget to multiply by derivative of inner function
- 3. Integration: Remember the constant of integration
- 4. Definite integrals: Apply limits correctly
- 5. Complex numbers:  $i^2 = -1$ , not +1
- 6. Differential equations: Don't forget integrating factor for linear DE
- 7. **Parametric differentiation**: Use  $\frac{dy/dt}{dx/dt}$ , not  $\frac{dt/dy}{dt/dx}$

# **Exam Tips**

#### **Time Management**

- Q.1 (MCQs): Spend 15-20 minutes maximum
- Short answers: 3-4 minutes per question
- Long answers: 8-10 minutes per question

• Keep 10 minutes for final review

#### Strategy

- 1. Read all questions first to identify easy ones
- 2. Attempt easy questions first to build confidence
- 3. Show all steps clearly for partial marks
- 4. Check units in application problems
- 5. Verify answers where possible (especially in matrix problems)

#### **During Exam**

- Write clearly and organize solutions
- Draw diagrams where helpful
- State formulas before using them
- Don't panic if stuck on one question move to next
- Use remaining time to review and check calculations

#### Good Luck with your exams!