Q.1 [14 marks]

Fill in the blanks using appropriate choice from the given options.

Q1.1 [1 mark]

If A = $\begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ then adj.A = __.

Answer: (d) $\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$

Solution:
For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, adj.A = $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$adj.\ A = \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix}$

Q1.2 [1 mark]

If A is 2×3 and B is 3×4 matrices then AB is __ matrix

Answer: (b) 2×4

Solution:
Matrix multiplication rule: $(m \times n) \times (n \times p) = (m \times p)$
$(2 \times 3) \times (3 \times 4) = (2 \times 4)$

Q1.3 [1 mark]

If $\begin{bmatrix} 0 & x \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -2 & 4 \end{bmatrix}$ then x = __

Answer: (b) 4

Solution:
Comparing corresponding elements: $x = 4$

Q1.4 [1 mark]

If A is non singular matrix then __

Answer: (d) $|A| \neq 0$

Solution:
A matrix is non-singular if its determinant is non-zero.

Q1.5 [1 mark]

$\frac{d}{dx}(e^{-\log x}) = \_\_\_\_$

Answer: (d) x

Solution:
$e^{-\log x} = e^{\log x^{-1}} = x^{-1} = \frac{1}{x}$
$\frac{d}{dx}(\frac{1}{x}) = -\frac{1}{x^2}$

Q1.6 [1 mark]

If $f(x) = \log\sqrt{x^2+1}$, then $f'(0) =$ ___

Answer: (a) 0

Solution:
$f(x) = \frac{1}{2}\log(x^2+1)$
$f'(x) = \frac{1}{2} \cdot \frac{2x}{x^2+1} = \frac{x}{x^2+1}$
$f'(0) = \frac{0}{0+1} = 0$

Q1.7 [1 mark]

If $x = \sec\theta + \tan\theta$ and $y = \sec\theta - \tan\theta$ then $\frac{dy}{dx} =$ ___

Answer: (d) 1

Solution:
$xy = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = \sec^2\theta - \tan^2\theta = 1$
Differentiating: $x\frac{dy}{dx} + y = 0$
$\frac{dy}{dx} = -\frac{y}{x}$

Q1.8 [1 mark]

$\int e^x(\sin x + \cos x)dx =$ ____

Answer: (b) $e^x \sin x + c$

Solution:
Using integration by parts or standard result:
$\int e^x(\sin x + \cos x)dx = e^x \sin x + c$

Q1.9 [1 mark]

$\int_{-1}^{1} x^2 + 1dx =$ _

Answer: (d) $\frac{8}{3}$

Solution:
$\int_{-1}^{1}(x^2+1)dx = [\frac{x^3}{3} + x]_{-1}^{1}$
$= (\frac{1}{3} + 1) - (\frac{-1}{3} - 1) = \frac{8}{3}$

Q1.10 [1 mark]

$\int \cot x dx =$ ___ **+ c**

Answer: (a) $\log|\sin x|$

Solution:
$\int \cot x dx = \int \frac{\cos x}{\sin x} dx = \log|\sin x| + c$

Q1.11 [1 mark]

The order & degree of the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 3y = 0$ are respectively___ and ___

Answer: (a) 2, 1

Solution:
Order = highest order derivative = 2
Degree = power of highest order derivative = 1

Q1.12 [1 mark]

The integrating factor for the differential equation $\frac{dy}{dx} + \frac{y}{x} = x$ is

Answer: (b) x

Solution:
For $\frac{dy}{dx} + P(x)y = Q(x)$, where $P(x) = \frac{1}{x}$
I.F. = $e^{\int P(x)dx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$

Q1.13 [1 mark]

$i + i^2 + i^3 + i^4 = \_$

Answer: (d) 0

Solution:
$i + i^2 + i^3 + i^4 = i + (-1) + (-i) + 1 = 0$

Q1.14 [1 mark]

arg(-1) = ___

Answer: (a) π

Solution:
$-1 = \cos\pi + i\sin\pi$, so $\arg(-1) = \pi$

Q.2(a) [6 marks]

Attempt any two.

Q2(a).1 [3 marks]

If $A = \begin{bmatrix} 1 & 2 \\ -3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 6 \\ -2 & 3 \end{bmatrix}$ then find matrix X from equation 3(X+B) + 5A = 0

Solution:
$3(X+B)+5A=0$
$3X+3B+5A=0$
$3X=-3B-5A$
$X=-B-\frac{5A}{3}$

$$5A=5\begin{bmatrix}1 & 2\\-3 & 2\end{bmatrix}=\begin{bmatrix}5 & 10\\-15 & 10\end{bmatrix}$$

$$X=-\begin{bmatrix}5 & 6\\-2 & 3\end{bmatrix}-\frac{1}{3}\begin{bmatrix}5 & 10\\-15 & 10\end{bmatrix}$$

$$X=\begin{bmatrix}-5 & -6\\2 & -3\end{bmatrix}-\begin{bmatrix}\frac{5}{3} & \frac{10}{3}\\-5 & \frac{10}{3}\end{bmatrix}$$

$$X=\begin{bmatrix}-\frac{20}{3} & -\frac{28}{3}\\7 & -\frac{19}{3}\end{bmatrix}$$

Q2(a).2 [3 marks]

If $A=\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$ **then Prove that** $A^2-4A-5I=0$

Solution:

$$A^2=\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}=\begin{bmatrix}5 & 4\\4 & 5\end{bmatrix}$$

$$4A=4\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}=\begin{bmatrix}4 & 8\\8 & 4\end{bmatrix}$$

$$5I=5\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}$$

$$A^2-4A-5I=\begin{bmatrix}5 & 4\\4 & 5\end{bmatrix}-\begin{bmatrix}4 & 8\\8 & 4\end{bmatrix}-\begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}$$

$$=\begin{bmatrix}0 & -4\\-4 & 0\end{bmatrix}-\begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

Hence proved.

Q2(a).3 [3 marks]

Solve differential equation $\frac{dy}{dx}=(x+y)^2$

Solution:
Let $v=x+y$, then $\frac{dv}{dx}=1+\frac{dy}{dx}$
$\frac{dy}{dx}=\frac{dv}{dx}-1$

Substituting: $\frac{dv}{dx}-1=v^2$
$\frac{dv}{dx}=v^2+1$
$\frac{dv}{v^2+1}=dx$

Integrating: $\int \frac{dv}{v^2+1} = \int dx$
$\tan^{-1} v = x + c$
$\tan^{-1}(x+y) = x + c$
$x + y = \tan(x+c)$
$y = \tan(x+c) - x$

Q.2(b) [8 marks]

Attempt any two.

Q2(b).1 [4 marks]

If $A = \begin{bmatrix} 3 & -1 \\ 4 & 1 \\ 5 & 0 \end{bmatrix}$ **then find** A^{-1}

Solution:
This is a 3×2 matrix, which is non-square. Inverse doesn't exist for non-square matrices.

Alternative interpretation - if it's $\begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$**:**

Using adjoint method:
$|A| = 3(1-0) + 1(4+5) + 2(0-5) = 3 + 9 - 10 = 2$

Calculate cofactors and adjoint, then $A^{-1} = \frac{1}{|A|} \times adj(A)$

Q2(b).2 [4 marks]

Solve Equation 3X-2Y=8 and 5X+4Y=6 using matrices method.

Solution:
$\begin{bmatrix} 3 & -2 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$

$|A| = 3(4) - (-2)(5) = 12 + 10 = 22$

$$A^{-1} = \frac{1}{22} \begin{bmatrix} 4 & 2 \\ -5 & 3 \end{bmatrix}$$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{22} \begin{bmatrix} 4 & 2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 6 \end{bmatrix}$
$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{22} \begin{bmatrix} 32+12 \\ -40+18 \end{bmatrix} = \frac{1}{22} \begin{bmatrix} 44 \\ -22 \end{bmatrix}$

$X = 2, Y = -1$

Q2(b).3 [4 marks]

If $M = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$, $N = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$ **then Prove that** $(MN)^T = N^T M^T$

Solution:
$MN = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 11 \\ 2 & 1 \end{bmatrix}$

$(MN)^T = \begin{bmatrix} 12 & 2 \\ 11 & 1 \end{bmatrix}$

$M^T = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$, $N^T = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$

$N^T M^T = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ 11 & 1 \end{bmatrix}$

Hence $(MN)^T = N^T M^T$ is proved.

Q.3(a) [6 marks]

Attempt any two.

Q3(a).1 [3 marks]

Differentiate $\sqrt{x}$ using the definition.

Solution:
$f(x) = \sqrt{x} = x^{1/2}$

Using definition: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$

Rationalizing: $f'(x) = \lim_{h\to 0} \frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim_{h\to 0} \frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$

Q3(a).2 [3 marks]

If $y = \log(x + \sqrt{1+x^2})$ then Find $\frac{dy}{dx}$

Solution:
$y = \log(x + \sqrt{1+x^2})$

$\frac{dy}{dx} = \frac{1}{x+\sqrt{1+x^2}} \cdot \frac{d}{dx}(x + \sqrt{1+x^2})$

$\frac{d}{dx}(x + \sqrt{1+x^2}) = 1 + \frac{1}{2\sqrt{1+x^2}} \cdot 2x = 1 + \frac{x}{\sqrt{1+x^2}}$

$= \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}$

$\frac{dy}{dx} = \frac{1}{x+\sqrt{1+x^2}} \cdot \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}$

$= \frac{1}{\sqrt{1+x^2}}$

Q3(a).3 [3 marks]

$\int \frac{4+3\cos x}{\sin^2 x} dx$

Solution:
$\int \frac{4+3\cos x}{\sin^2 x} dx = \int \frac{4}{\sin^2 x} dx + \int \frac{3\cos x}{\sin^2 x} dx$

$= 4 \int \csc^2 x dx + 3 \int \frac{\cos x}{\sin^2 x} dx$

$= -4 \cot x + 3 \int \sin^{-2} x \cos x dx$

For the second integral, let $u = \sin x$, $du = \cos x dx$
$3 \int u^{-2} du = 3(-u^{-1}) = -\frac{3}{\sin x}$

$\int \frac{4+3\cos x}{\sin^2 x} dx = -4 \cot x - 3 \csc x + c$

Q.3(b) [8 marks]

Attempt any two.

Q3(b).1 [4 marks]

If $y = \log(\sin x)$ then prove that $\frac{d^2y}{dx^2} + (\frac{dy}{dx})^2 + 1 = 0$

Solution:
$y = \log(\sin x)$

$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x = \cot x$

$\frac{d^2y}{dx^2} = \frac{d}{dx}(\cot x) = -\csc^2 x$

Now, $\frac{d^2y}{dx^2} + (\frac{dy}{dx})^2 + 1 = -\csc^2 x + \cot^2 x + 1$

Using identity: $\csc^2 x - \cot^2 x = 1$
$-\csc^2 x + \cot^2 x + 1 = -(\csc^2 x - \cot^2 x) = -1 + 1 = 0$

Hence proved.

Q3(b).2 [4 marks]

If $x + y = \sin(xy)$ then Find $\frac{dy}{dx}$

Solution:
$x + y = \sin(xy)$

Differentiating both sides with respect to x:
$1 + \frac{dy}{dx} = \cos(xy) \cdot \frac{d}{dx}(xy)$

$1 + \frac{dy}{dx} = \cos(xy) \cdot (y + x\frac{dy}{dx})$

$1 + \frac{dy}{dx} = y\cos(xy) + x\cos(xy)\frac{dy}{dx}$

$1 + \frac{dy}{dx} - x\cos(xy)\frac{dy}{dx} = y\cos(xy)$

$\frac{dy}{dx}(1 - x\cos(xy)) = y\cos(xy) - 1$
$\frac{dy}{dx} = \frac{y\cos(xy)-1}{1-x\cos(xy)}$

Q3(b).3 [4 marks]

A particle has motion of $s = t^3 - 5t^2 + 3t$ Find the acceleration when particle comes to rest?

Solution:
Given: $s = t^3 - 5t^2 + 3t$

Velocity: $v = \frac{ds}{dt} = 3t^2 - 10t + 3$

Acceleration: $a = \frac{dv}{dt} = 6t - 10$

At rest, $v = 0$:
$3t^2 - 10t + 3 = 0$

Using quadratic formula: $t = \frac{10\pm\sqrt{100-36}}{6} = \frac{10\pm8}{6}$

$t = 3$ or $t = \frac{1}{3}$

At $t = 3$: $a = 6(3) - 10 = 8$
At $t = \frac{1}{3}$: $a = 6(\frac{1}{3}) - 10 = -8$

The accelerations are 8 and -8 respectively.

Q.4(a) [6 marks]

Attempt any two.

Q4(a).1 [3 marks]

$\int x\sin x dx$

Solution:
Using integration by parts: $\int udv = uv - \int vdu$

Let $u = x$, $dv = \sin x dx$
$du = dx$, $v = -\cos x$

$\int x\sin x dx = x(-\cos x) - \int(-\cos x)dx$
$= -x\cos x + \int\cos x dx$
$= -x\cos x + \sin x + c$

Q4(a).2 [3 marks]

$\int \frac{2x+1}{(x+1)(x-3)}dx$

Solution:
Using partial fractions:
$\frac{2x+1}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$

$2x + 1 = A(x-3) + B(x+1)$

At $x = -1$: $-2 + 1 = A(-4) \Rightarrow A = \frac{1}{4}$
At $x = 3$: $6 + 1 = B(4) \Rightarrow B = \frac{7}{4}$

$\int \frac{2x+1}{(x+1)(x-3)} dx = \frac{1}{4} \int \frac{1}{x+1} dx + \frac{7}{4} \int \frac{1}{x-3} dx$

$= \frac{1}{4} \log|x+1| + \frac{7}{4} \log|x-3| + c$

Q4(a).3 [3 marks]

Find square root of complex number $z = 7 + 24i$

Solution:
Let $\sqrt{7+24i} = a + bi$

$(a+bi)^2 = 7 + 24i$
$a^2 - b^2 + 2abi = 7 + 24i$

Comparing: $a^2 - b^2 = 7$ and $2ab = 24$
From second equation: $b = \frac{12}{a}$

Substituting: $a^2 - \frac{144}{a^2} = 7$
$a^4 - 7a^2 - 144 = 0$

Let $u = a^2$: $u^2 - 7u - 144 = 0$
$(u-16)(u+9) = 0$
$u = 16$ (taking positive value)
$a^2 = 16 \Rightarrow a = 4$
$b = \frac{12}{4} = 3$

Therefore: $\sqrt{7+24i} = 4 + 3i$ or $-(4+3i)$

Q.4(b) [8 marks]

Attempt any two.

Q4(b).1 [4 marks]

$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$

Solution:
Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$

Using property: $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\sin(\pi/2-x)}+\sqrt{\cos(\pi/2-x)}} dx$

$= \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$

Adding both expressions:
$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx = \int_0^{\pi/2} 1 dx = \frac{\pi}{2}$

Therefore: $I = \frac{\pi}{4}$

Q4(b).2 [4 marks]

Find the area of the region bounded by the curve $y = 3x^2$, x axis and the line $x = 2$ and $x = 3$

Solution:
Area = $\int_2^3 y dx = \int_2^3 3x^2 dx$

$= 3\int_2^3 x^2 dx = 3[\frac{x^3}{3}]_2^3$

$= [x^3]_2^3 = 3^3 - 2^3 = 27 - 8 = 19$

Area = 19 square units

Q4(b).3 [4 marks]

Simplify $\frac{(\cos 2\theta + i\sin 2\theta)^{-3} \cdot (\cos 3\theta - i\sin 3\theta)^2}{(\cos 2\theta - i\sin 2\theta)^{-7} \cdot (\cos 5\theta - i\sin 5\theta)^3}$

Solution:
Using Euler's formula: $\cos\theta + i\sin\theta = e^{i\theta}$

$(\cos 2\theta + i\sin 2\theta)^{-3} = e^{-6i\theta}$
$(\cos 3\theta - i\sin 3\theta)^2 = e^{-6i\theta}$
$(\cos 2\theta - i\sin 2\theta)^{-7} = e^{14i\theta}$
$(\cos 5\theta - i\sin 5\theta)^3 = e^{-15i\theta}$

Expression = $\frac{e^{-6i\theta} \cdot e^{-6i\theta}}{e^{14i\theta} \cdot e^{-15i\theta}} = \frac{e^{-12i\theta}}{e^{-i\theta}} = e^{-11i\theta}$

$= \cos(-11\theta) + i\sin(-11\theta) = \cos(11\theta) - i\sin(11\theta)$

Q.5(a) [6 marks]

Attempt any two.

Q5(a).1 [3 marks]

Convert $\frac{4+2i}{(3+2i)(5-3i)}$ in a+ib form.

Solution:
First, simplify the denominator:
$(3+2i)(5-3i) = 15 - 9i + 10i - 6i^2 = 15 + i + 6 = 21 + i$

Now: $\frac{4+2i}{21+i}$

Multiply by conjugate: $\frac{4+2i}{21+i} \cdot \frac{21-i}{21-i}$

$= \frac{(4+2i)(21-i)}{(21+i)(21-i)} = \frac{84-4i+42i-2i^2}{441-i^2}$

$= \frac{84+38i+2}{441+1} = \frac{86+38i}{442} = \frac{43+19i}{221}$

Q5(a).2 [3 marks]

Convert $z = 1 - \sqrt{3}i$ in polar form.

Solution:
$z = 1 - \sqrt{3}i$

$|z| = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1+3} = 2$

$\arg(z) = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -\frac{\pi}{3}$ (since z is in 4th quadrant)

Therefore: $z = 2(\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})) = 2e^{-i\pi/3}$

Q5(a).3 [3 marks]

Prove that $(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n(\frac{\theta}{2})\cos(\frac{n\theta}{2})$

Solution:
$1 + \cos\theta + i\sin\theta = 1 + e^{i\theta} = 1 + \cos\theta + i\sin\theta$

Using identity: $1 + \cos\theta = 2\cos^2(\frac{\theta}{2})$

$1 + \cos\theta + i\sin\theta = 2\cos^2(\frac{\theta}{2}) + 2i\sin(\frac{\theta}{2})\cos(\frac{\theta}{2})$

$= 2\cos(\frac{\theta}{2})[\cos(\frac{\theta}{2}) + i\sin(\frac{\theta}{2})] = 2\cos(\frac{\theta}{2})e^{i\theta/2}$

Similarly: $1 + \cos\theta - i\sin\theta = 2\cos(\frac{\theta}{2})e^{-i\theta/2}$

$(1+\cos\theta + i\sin\theta)^n = 2^n\cos^n(\frac{\theta}{2})e^{in\theta/2}$

$(1+\cos\theta - i\sin\theta)^n = 2^n\cos^n(\frac{\theta}{2})e^{-in\theta/2}$

Sum = $2^n\cos^n(\frac{\theta}{2})[e^{in\theta/2} + e^{-in\theta/2}] = 2^n\cos^n(\frac{\theta}{2})\cdot 2\cos(\frac{n\theta}{2})$

$= 2^{n+1}\cos^n(\frac{\theta}{2})\cos(\frac{n\theta}{2})$

Hence proved.

Q.5(b) [8 marks]

Attempt any two.

Q5(b).1 [4 marks]

Solve differential equation $x\log x\frac{dy}{dx} + y = \log x^2$

Solution:
$x\log x\frac{dy}{dx} + y = 2\log x$

Dividing by $x\log x$:
$\frac{dy}{dx} + \frac{y}{x\log x} = \frac{2}{x}$

This is a linear differential equation: $\frac{dy}{dx} + P(x)y = Q(x)$

Where $P(x) = \frac{1}{x\log x}$ and $Q(x) = \frac{2}{x}$

Integrating Factor: $e^{\int P(x)dx} = e^{\int\frac{1}{x\log x}dx}$

Let $u = \log x$, then $du = \frac{1}{x}dx$
$\int \frac{1}{x \log x}dx = \int \frac{1}{u}du = \log u = \log(\log x)$

I.F. = $e^{\log(\log x)} = \log x$

Solution: $y \cdot \log x = \int \frac{2}{x} \cdot \log x dx$

$= 2\int \frac{\log x}{x}dx = 2 \cdot \frac{(\log x)^2}{2} = (\log x)^2$

Therefore: $y = \frac{(\log x)^2}{\log x} = \log x$

Q5(b).2 [4 marks]

Solve differential equation $\frac{dy}{dx} - \frac{y}{x} = e^x$

Solution:
This is a linear differential equation: $\frac{dy}{dx} + P(x)y = Q(x)$

Where $P(x) = -\frac{1}{x}$ and $Q(x) = e^x$

Integrating Factor: $e^{\int P(x)dx} = e^{\int -\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$

Solution: $y \cdot \frac{1}{x} = \int e^x \cdot \frac{1}{x}dx$

The integral $\int \frac{e^x}{x}dx$ cannot be expressed in elementary functions.

Alternative approach - assuming it's $\frac{dy}{dx} + \frac{y}{x} = e^x$:

I.F. = $e^{\int \frac{1}{x}dx} = e^{\log x} = x$

$y \cdot x = \int e^x \cdot x dx$

Using integration by parts:
$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x = e^x(x-1)$

Therefore: $xy = e^x(x-1) + c$
$y = \frac{e^x(x-1)+c}{x}$

Q5(b).3 [4 marks]

Solve differential equation $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$, $y(\frac{\pi}{4}) = \frac{\pi}{4}$

Solution:
$\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

Rearranging: $\frac{\sec^2 x}{\tan x}dx + \frac{\sec^2 y}{\tan y}dy = 0$

$\frac{\cos x}{\sin x \cos^2 x}dx + \frac{\cos y}{\sin y \cos^2 y}dy = 0$

$\frac{1}{\sin x \cos x}dx + \frac{1}{\sin y \cos y}dy = 0$

$\frac{2}{\sin 2x}dx + \frac{2}{\sin 2y}dy = 0$

$\csc(2x)dx + \csc(2y)dy = 0$

Integrating: $\int \csc(2x)dx + \int \csc(2y)dy = c$

$-\frac{1}{2}\log|\csc(2x)+\cot(2x)| - \frac{1}{2}\log|\csc(2y)+\cot(2y)| = c$

$\log|\csc(2x)+\cot(2x)| + \log|\csc(2y)+\cot(2y)| = -2c = k$

$|\csc(2x)+\cot(2x)| \cdot |\csc(2y)+\cot(2y)| = e^k$

Using initial condition $y(\frac{\pi}{4}) = \frac{\pi}{4}$:
At $x = \frac{\pi}{4}$, $y = \frac{\pi}{4}$

$|\csc(\frac{\pi}{2})+\cot(\frac{\pi}{2})| \cdot |\csc(\frac{\pi}{2})+\cot(\frac{\pi}{2})| = |1+0| \cdot |1+0| = 1$

Therefore: $(\csc(2x)+\cot(2x))(\csc(2y)+\cot(2y)) = 1$

Complete Formula Cheat Sheet

Matrix Operations

Operation	Formula
Adjoint (2×2)	If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $adj(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
Inverse	$A^{-1} = \frac{1}{\lvert A\rvert} \times adj(A)$
Matrix Multiplication	$(AB)_{ij} = \sum_k A_{ik}B_{kj}$
Transpose Property	$(AB)^T = B^T A^T$

Differentiation

Function	Derivative
x^n	nx^{n-1}
$\log x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
Chain Rule	$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$
Product Rule	$(uv)' = u'v + uv'$
Quotient Rule	$(\frac{u}{v})' = \frac{u'v-uv'}{v^2}$

Integration

Function	Integral
x^n	$\frac{x^{n+1}}{n+1} + c$
$\frac{1}{x}$	$\log\lvert x\rvert + c$
e^x	$e^x + c$
$\sin x$	$-\cos x + c$
$\cos x$	$\sin x + c$
$\sec^2 x$	$\tan x + c$
$\csc^2 x$	$-\cot x + c$
Integration by Parts	$\int udv = uv - \int vdu$

Differential Equations

Type	Method	Solution
Variable Separable	$\frac{dy}{dx} = f(x)g(y)$	$\int \frac{dy}{g(y)} = \int f(x)dx$
Linear DE	$\frac{dy}{dx} + Py = Q$	$y \cdot I.F. = \int Q \cdot I.F. dx$
Integrating Factor	I.F. = $e^{\int Pdx}$	-

Complex Numbers

Operation	Formula
Modulus	$\lvert a + bi\rvert = \sqrt{a^2 + b^2}$
Argument	$\arg(z) = \tan^{-1}(\frac{b}{a})$
Polar Form	$z = r(\cos\theta + i\sin\theta) = re^{i\theta}$
Powers	$i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1$
De Moivre's	$(r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta + i\sin n\theta)$

Problem-Solving Strategies

For Matrix Problems:

1. **Check dimensions** first for multiplication
2. **Use determinant** to check if inverse exists
3. **Apply properties** like $(AB)^T = B^T A^T$

4. **Substitute and verify** your answers

For Differentiation:

1. **Identify the type** (composite, product, quotient)
2. **Apply appropriate rule** systematically
3. **Simplify step by step**
4. **Check using basic derivatives**

For Integration:

1. **Look for standard forms** first
2. **Try substitution** if composite function
3. **Use integration by parts** for products
4. **Apply partial fractions** for rational functions

For Differential Equations:

1. **Identify the type** (separable, linear, etc.)
2. **Find integrating factor** for linear DEs
3. **Separate variables** when possible
4. **Apply initial conditions** to find constants

Common Mistakes to Avoid

Matrix Operations:

- **Wrong dimension calculation** in multiplication
- **Forgetting to transpose** in $(AB)^T = B^T A^T$
- **Not checking if matrix is invertible** before finding inverse

Differentiation:

- **Missing chain rule** in composite functions
- **Sign errors** in trigonometric derivatives
- **Forgetting product rule** in multiplied functions

Integration:

- **Wrong limits** in definite integrals
- **Missing constant of integration**
- **Incorrect substitution** bounds

Complex Numbers:

- **Wrong quadrant** in argument calculation
- **Modulus calculation errors**
- **Forgetting to rationalize** denominators

Exam Tips

Time Management:

- **Attempt Q.1 first** (14 marks, quick fill-ups)
- **Choose easier sub-questions** in each section
- **Leave difficult calculations** for the end

Answer Presentation:

- **Show all steps** clearly
- **Box final answers**
- **Use proper mathematical notation**
- **Draw diagrams** where helpful

Verification:

- **Check dimensions** in matrix problems
- **Verify differentiation** by differentiating your answer
- **Substitute back** in differential equations
- **Check modulus and argument** for complex numbers

Key Formulas to Remember:

- **Matrix inverse formula**
- **Integration by parts**
- **Linear DE solution method**
- **Complex number polar form**
- **Standard derivatives and integrals**

Remember: Practice is key to mastering these concepts. Work through similar problems and focus on understanding the underlying principles rather than just memorizing formulas.