Q.1 Fill in the blanks [14 marks]

Q1.1 [1 mark]

Order of the matrix $\begin{bmatrix} 2 & 5 \\ 7 & 8 \end{bmatrix}$ is ____

Answer: (d) 2×2

Solution:
The matrix has 2 rows and 2 columns, so its order is 2×2.

Q1.2 [1 mark]

$\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 5 \\ 5 & 8 \end{bmatrix} =$ ____

Answer: (a) $\begin{bmatrix} 5 & 8 \\ 11 & 10 \end{bmatrix}$

Solution:
$\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 5 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 4+1 & 3+5 \\ 6+5 & 2+8 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ 11 & 10 \end{bmatrix}$

Q1.3 [1 mark]

Which of the following is a square matrix?

Answer: (c) $\begin{bmatrix} 1 & 3 \\ 5 & 4 \end{bmatrix}$

Solution:
A square matrix has equal number of rows and columns. Only option (c) has 2×2 dimensions.

Q1.4 [1 mark]

If $A = [3]$ and $B = [4]$ then $A \cdot B =$ ____

Answer: (b) 12

Solution:
$A \cdot B = [3] \times [4] = [3 \times 4] = [12] = 12$

Q1.5 [1 mark]

$\frac{d}{dx} \sin x =$ ____

Answer: (d) $\cos x$

Solution:
The derivative of $\sin x$ is $\cos x$.

Q1.6 [1 mark]

If $f(x) = xe^x$ then $f'(0) =$ ____

Answer: (b) 1

Solution:
Using product rule: $f'(x) = \frac{d}{dx}(xe^x) = e^x + xe^x = e^x(1+x)$
$f'(0) = e^0(1+0) = 1 \times 1 = 1$

Q1.7 [1 mark]

If $y = x^2$ then $\frac{d^2y}{dx^2} =$ ____

Answer: (b) 2

Solution:
$y = x^2$
$\frac{dy}{dx} = 2x$
$\frac{d^2y}{dx^2} = 2$

Q1.8 [1 mark]

$\int \cos x dx =$ ____ $+c$

Answer: (a) $\sin x$

Solution:
$\int \cos x dx = \sin x + c$

Q1.9 [1 mark]

$\int_0^1 x dx =$ ____

Answer: (c) $\frac{1}{2}$

Solution:
$\int_0^1 x dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$

Q1.10 [1 mark]

$\int \frac{1}{1+x^2} dx =$ ____ $+c$

Answer: (a) $\tan^{-1} x$

Solution:
$\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Q1.11 [1 mark]

Order of differential equation $x \sin y + xy = x$ is ____

Answer: (b) 1

Solution:
The equation can be written as $\frac{dy}{dx} = \frac{1-xy}{\sin y}$. The highest order derivative is first order.

Q1.12 [1 mark]

Integration factor of $\frac{dy}{dx} + y = x$ **is ____**

Answer: (d) e^x

Solution:
For $\frac{dy}{dx} + Py = Q$, integration factor $= e^{\int Pdx} = e^{\int 1dx} = e^x$

Q1.13 [1 mark]

$i^2 =$ ____

Answer: (b) -1

Solution:
By definition, $i^2 = -1$

Q1.14 [1 mark]

$(2+3i)(2-3i) =$ ____

Answer: (c) 13

Solution:
$(2+3i)(2-3i) = 2^2 - (3i)^2 = 4 - 9i^2 = 4 - 9(-1) = 4 + 9 = 13$

Q.2(A) Attempt any two [6 marks]

Q2.1(A)(1) [3 marks]

If $A = \begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 8 \\ 4 & 6 \end{bmatrix}$ **and** $C = \begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix}$ **then find** $2A + 3B - C$

Solution:

$$2A = 2\begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ -2 & 6 \end{bmatrix}$$

$$3B = 3\begin{bmatrix} 5 & 8 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 15 & 24 \\ 12 & 18 \end{bmatrix}$$

$$2A + 3B = \begin{bmatrix} 4 & 10 \\ -2 & 6 \end{bmatrix} + \begin{bmatrix} 15 & 24 \\ 12 & 18 \end{bmatrix} = \begin{bmatrix} 19 & 34 \\ 10 & 24 \end{bmatrix}$$

$$2A + 3B - C = \begin{bmatrix} 19 & 34 \\ 10 & 24 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 15 & 32 \\ 9 & 19 \end{bmatrix}$$

Q2.1(A)(2) [3 marks]

If $M = \begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix}$ and $N = \begin{bmatrix} 6 & 9 \\ 0 & 5 \end{bmatrix}$ then prove that $(M+N)^T = M^T + N^T$

Solution:

$$M + N = \begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} 6 & 9 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 13 \\ 3 & 12 \end{bmatrix}$$

$$(M+N)^T = \begin{bmatrix} 7 & 3 \\ 13 & 12 \end{bmatrix}$$

$M^T = \begin{bmatrix} 1 & 3 \\ 4 & 7 \end{bmatrix}$, $N^T = \begin{bmatrix} 6 & 0 \\ 9 & 5 \end{bmatrix}$

$$M^T + N^T = \begin{bmatrix} 1 & 3 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 9 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 13 & 12 \end{bmatrix}$$

Hence, $(M+N)^T = M^T + N^T$ is proved.

Q2.1(A)(3) [3 marks]

Solve differential equation: $x\frac{dy}{dx} + y = xy$

Solution:

$x\frac{dy}{dx} + y = xy$
$\frac{dy}{dx} + \frac{y}{x} = y$
$\frac{dy}{dx} = y - \frac{y}{x} = y\left(1 - \frac{1}{x}\right) = y\left(\frac{x-1}{x}\right)$

Separating variables:
$\frac{dy}{y} = \frac{x-1}{x}dx$

Integrating:
$\ln|y| = \int \frac{x-1}{x}dx = \int \left(1 - \frac{1}{x}\right)dx = x - \ln|x| + C$

$y = Ae^{x - \ln|x|} = A\frac{e^x}{x}$

Q.2(B) Attempt any two [8 marks]

Q2.1(B)(1) [4 marks]

Solve equations $2x + 3y = 8$, $3x + 4y = 11$ using matrix method

Solution:

Writing in matrix form: $AX = B$

$$\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$$

Finding A^{-1}:

$|A| = 2(4) - 3(3) = 8 - 9 = -1$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 8 \\ 11 \end{bmatrix} = \begin{bmatrix} -32 + 33 \\ 24 - 22 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Therefore: $x = 1, y = 2$

Q2.1(B)(2) [4 marks]

If $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ **then prove that** $(AB)^T = B^T A^T$

Solution:
$AB = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 1 & 6 \end{bmatrix}$

$(AB)^T = \begin{bmatrix} 3 & 1 \\ 8 & 6 \end{bmatrix}$

$A^T = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$, $B^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$B^T A^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 6 \end{bmatrix}$

Hence, $(AB)^T = B^T A^T$ is proved.

Q2.1(B)(3) [4 marks]

If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ **then prove that** $A^2 - 4A + 7I = O$

Solution:
$A^2 = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$

$4A = 4 \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix}$

$7I = 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$A^2 - 4A + 7I = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

Hence proved.

Q.3(A) Attempt any two [6 marks]

Q3.1(A)(1) [3 marks]

Find derivative of $f(x) = e^x$ **using definition of differentiation**

Solution:
Using definition: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h}$

$= \lim_{h \to 0} \frac{e^x(e^h - 1)}{h} = e^x \lim_{h \to 0} \frac{e^h - 1}{h}$

Since $\lim_{h\to 0} \frac{e^h-1}{h} = 1$

Therefore: $f'(x) = e^x$

Q3.1(A)(2) [3 marks]

If $y = \log(\sin x)$ **then find** $\frac{dy}{dx}$

Solution:
$y = \log(\sin x)$

Using chain rule:
$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x) = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \cot x$

Q3.1(A)(3) [3 marks]

Evaluate: $\int \left(4x^3 + 3x^2 + \frac{2}{x}\right) dx$

Solution:
$\int \left(4x^3 + 3x^2 + \frac{2}{x}\right) dx$

$= \int 4x^3 dx + \int 3x^2 dx + \int \frac{2}{x} dx$

$= 4 \cdot \frac{x^4}{4} + 3 \cdot \frac{x^3}{3} + 2\ln|x| + C$

$= x^4 + x^3 + 2\ln|x| + C$

Q.3(B) Attempt any two [8 marks]

Q3.1(B)(1) [4 marks]

If $y = e^{\tan x} + \log(\sin x)$ **then find** $\frac{dy}{dx}$

Solution:
$y = e^{\tan x} + \log(\sin x)$

$\frac{dy}{dx} = \frac{d}{dx}[e^{\tan x}] + \frac{d}{dx}[\log(\sin x)]$

For first term: $\frac{d}{dx}[e^{\tan x}] = e^{\tan x} \cdot \sec^2 x$

For second term: $\frac{d}{dx}[\log(\sin x)] = \frac{1}{\sin x} \cdot \cos x = \cot x$

Therefore: $\frac{dy}{dx} = e^{\tan x} \sec^2 x + \cot x$

Q3.1(B)(2) [4 marks]

The equation of motion of a particle is $s = t^4 + 3t$**. Find its velocity and acceleration at** $t = 2$ **sec**

Solution:
Given: $s = t^4 + 3t$

Velocity: $v = \frac{ds}{dt} = 4t^3 + 3$

At $t = 2$: $v = 4(2)^3 + 3 = 4(8) + 3 = 32 + 3 = 35$ units/sec

Acceleration: $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 12t^2$

At $t = 2$: $a = 12(2)^2 = 12(4) = 48$ units/sec²

Q3.1(B)(3) [4 marks]

Find the maximum and minimum value of the function $f(x) = 2x^3 - 3x^2 - 12x + 5$

Solution:
$f(x) = 2x^3 - 3x^2 - 12x + 5$

$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x - 2)(x + 1)$

For critical points: $f'(x) = 0$
$6(x - 2)(x + 1) = 0$
$x = 2$ or $x = -1$

$f''(x) = 12x - 6$

At $x = -1$: $f''(-1) = 12(-1) - 6 = -18 < 0$ (Maximum)
At $x = 2$: $f''(2) = 12(2) - 6 = 18 > 0$ (Minimum)

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 5 = -2 - 3 + 12 + 5 = 12$ (Maximum)
$f(2) = 2(8) - 3(4) - 12(2) + 5 = 16 - 12 - 24 + 5 = -15$ (Minimum)

Maximum value: 12 at $x = -1$
Minimum value: -15 at $x = 2$

Q.4(A) Attempt any two [6 marks]

Q4.1(A)(1) [3 marks]

Evaluate: $\int xe^x dx$

Solution:
Using integration by parts: $\int udv = uv - \int vdu$

Let $u = x$, $dv = e^x dx$
Then $du = dx$, $v = e^x$

$\int xe^x dx = x \cdot e^x - \int e^x dx = xe^x - e^x + C = e^x(x - 1) + C$

Q4.1(A)(2) [3 marks]

Evaluate: $\int \frac{dx}{\sqrt{9-4x^2}}$

Solution:
$\int \frac{dx}{\sqrt{9-4x^2}} = \int \frac{dx}{\sqrt{9(1-\frac{4x^2}{9})}} = \int \frac{dx}{3\sqrt{1-(\frac{2x}{3})^2}}$

Let $\frac{2x}{3} = \sin\theta$, then $x = \frac{3\sin\theta}{2}$, $dx = \frac{3\cos\theta}{2}d\theta$

$= \int \frac{\frac{3\cos\theta}{2}d\theta}{3\sqrt{1-\sin^2\theta}} = \int \frac{\frac{3\cos\theta}{2}d\theta}{3\cos\theta} = \int \frac{1}{2}d\theta = \frac{\theta}{2} + C$

$= \frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right) + C$

Q4.1(A)(3) [3 marks]

Find complex conjugate of $\frac{1-i}{1+i}$

Solution:
$\frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{(1-i)^2}{1-i^2} = \frac{1-2i+i^2}{1-(-1)} = \frac{1-2i-1}{2} = \frac{-2i}{2} = -i$

Complex conjugate of $-i$ is $\overline{-i} = i$

Q.4(B) Attempt any two [8 marks]

Q4.1(B)(1) [4 marks]

Evaluate: $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$

Solution:
Let $I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$

Using property: $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi/2} \frac{\sqrt{\cos(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}}dx = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$

Adding both expressions:
$2I = \int_0^{\pi/2} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx = \int_0^{\pi/2} 1dx = \frac{\pi}{2}$

Therefore: $I = \frac{\pi}{4}$

Q4.1(B)(2) [4 marks]

Find the area of circle $x^2 + y^2 = a^2$ using integration

Solution:
For circle $x^2 + y^2 = a^2$, we have $y = \pm\sqrt{a^2 - x^2}$

Area of circle = $4\times$ Area in first quadrant
$= 4\int_0^a \sqrt{a^2 - x^2}dx$

Let $x = a\sin\theta$, $dx = a\cos\theta d\theta$
When $x = 0$, $\theta = 0$; when $x = a$, $\theta = \pi/2$

$= 4\int_0^{\pi/2} \sqrt{a^2 - a^2\sin^2\theta} \cdot a\cos\theta d\theta$
$= 4\int_0^{\pi/2} a\cos\theta \cdot a\cos\theta d\theta$
$= 4a^2\int_0^{\pi/2} \cos^2\theta d\theta$
$= 4a^2 \cdot \frac{\pi}{4} = \pi a^2$

Q4.1(B)(3) [4 marks]

Simplify: $\frac{(\cos 3\theta+i\sin 3\theta)^4\cdot(\cos\theta-i\sin\theta)^5}{(\cos 2\theta-i\sin 2\theta)^3\cdot(\cos 12\theta+i\sin 12\theta)}$

Solution:
Using De Moivre's theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

Numerator: $(\cos 3\theta + i\sin 3\theta)^4 \cdot (\cos\theta - i\sin\theta)^5$
$= (\cos 12\theta + i\sin 12\theta) \cdot (\cos(-5\theta) + i\sin(-5\theta))$
$= \cos(12\theta - 5\theta) + i\sin(12\theta - 5\theta)$
$= \cos 7\theta + i\sin 7\theta$

Denominator: $(\cos 2\theta - i\sin 2\theta)^3 \cdot (\cos 12\theta + i\sin 12\theta)$
$= (\cos(-6\theta) + i\sin(-6\theta)) \cdot (\cos 12\theta + i\sin 12\theta)$
$= \cos(-6\theta + 12\theta) + i\sin(-6\theta + 12\theta)$
$= \cos 6\theta + i\sin 6\theta$

Result: $\frac{\cos 7\theta + i\sin 7\theta}{\cos 6\theta + i\sin 6\theta} = \cos(7\theta - 6\theta) + i\sin(7\theta - 6\theta) = \cos\theta + i\sin\theta$

Q.5(A) Attempt any two [6 marks]

Q5.1(A)(1) [3 marks]

If $(3x - 7) + 2iy = 5y + (5 + x)i$ then find value of x and y

Solution:
$(3x - 7) + 2iy = 5y + (5 + x)i$

Comparing real and imaginary parts:
Real parts: $3x - 7 = 5y$ ... (1)
Imaginary parts: $2y = 5 + x$ ... (2)

From equation (2): $x = 2y - 5$ ... (3)

Substituting (3) in (1):
$3(2y - 5) - 7 = 5y$
$6y - 15 - 7 = 5y$
$6y - 22 = 5y$
$y = 22$

From (3): $x = 2(22) - 5 = 44 - 5 = 39$

Therefore: $x = 39, y = 22$

Q5.1(A)(2) [3 marks]

Convert $z = 1 + \sqrt{3}i$ into polar form

Solution:
$z = 1 + \sqrt{3}i$

Modulus: $|z| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1 + 3} = \sqrt{4} = 2$

Argument: $\arg(z) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

Polar form: $z = |z|(\cos\theta + i\sin\theta) = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Q5.1(A)(3) [3 marks]

Express $\frac{4+2i}{(3+2i)(5-3i)}$ in $a+ib$ form

Solution:
First, simplify denominator:
$(3+2i)(5-3i) = 15 - 9i + 10i - 6i^2 = 15 + i - 6(-1) = 15 + i + 6 = 21 + i$

$\frac{4+2i}{21+i} = \frac{(4+2i)(21-i)}{(21+i)(21-i)} = \frac{84-4i+42i-2i^2}{21^2-i^2} = \frac{84+38i+2}{441+1} = \frac{86+38i}{442}$

$= \frac{86}{442} + \frac{38}{442}i = \frac{43}{221} + \frac{19}{221}i$

Q.5(B) Attempt any two [8 marks]

Q5.1(B)(1) [4 marks]

Solve differential equation: $\frac{dy}{dx} + 2y = 3e^x$

Solution:
This is a first-order linear differential equation of the form $\frac{dy}{dx} + Py = Q$

Here: $P = 2$, $Q = 3e^x$

Integration factor: $\mu = e^{\int P dx} = e^{\int 2 dx} = e^{2x}$

Multiplying equation by μ:
$e^{2x}\frac{dy}{dx} + 2e^{2x}y = 3e^{2x} \cdot e^x = 3e^{3x}$

This gives: $\frac{d}{dx}(ye^{2x}) = 3e^{3x}$

Integrating both sides:
$ye^{2x} = \int 3e^{3x}dx = 3 \cdot \frac{e^{3x}}{3} + C = e^{3x} + C$

Therefore: $y = \frac{e^{3x}+C}{e^{2x}} = e^x + Ce^{-2x}$

Q5.1(B)(2) [4 marks]

Solve differential equation: $\frac{dy}{dx} = (x+y)^2$

Solution:
Let $v = x + y$, then $\frac{dv}{dx} = 1 + \frac{dy}{dx}$

So $\frac{dy}{dx} = \frac{dv}{dx} - 1$

Substituting in the original equation:
$\frac{dv}{dx} - 1 = v^2$
$\frac{dv}{dx} = v^2 + 1$

Separating variables:
$\frac{dv}{v^2+1} = dx$

Integrating both sides:
$\int \frac{dv}{v^2+1} = \int dx$
$\tan^{-1}(v) = x + C$
$v = \tan(x + C)$

Substituting back: $x + y = \tan(x + C)$
Therefore: $y = \tan(x + C) - x$

Q5.1(B)(3) [4 marks]

Solve differential equation: $\frac{dy}{dx} + \frac{y}{x} = e^x$, $y(0) = 2$

Solution:
This is a first-order linear differential equation: $\frac{dy}{dx} + \frac{y}{x} = e^x$

Here: $P = \frac{1}{x}$, $Q = e^x$

Integration factor: $\mu = e^{\int \frac{1}{x} dx} = e^{\ln|x|} = |x| = x$ (for $x > 0$)

Multiplying equation by $\mu = x$:
$x \frac{dy}{dx} + y = xe^x$

This gives: $\frac{d}{dx}(xy) = xe^x$

Integrating both sides using integration by parts:
$xy = \int xe^x dx$

For $\int xe^x dx$: Let $u = x$, $dv = e^x dx$
Then $du = dx$, $v = e^x$
$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x = e^x(x - 1)$

So: $xy = e^x(x - 1) + C$
$y = \frac{e^x(x-1)+C}{x}$

Using initial condition $y(0) = 2$:
This presents a problem as we have division by zero. The equation needs to be solved more carefully near $x = 0$.

For the general solution: $y = e^x \left(1 - \frac{1}{x}\right) + \frac{C}{x}$

Formula Cheat Sheet

Matrix Operations

- Matrix addition: $(A + B)_{ij} = A_{ij} + B_{ij}$
- Matrix multiplication: $(AB)_{ij} = \sum_k A_{ik} B_{kj}$
- Transpose: $(A^T)_{ij} = A_{ji}$
- Inverse of 2×2 matrix: $A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Differentiation Formulas

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- Product rule: $(uv)' = u'v + uv'$
- Chain rule: $\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$

Integration Formulas

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$)
- $\int \frac{1}{x} dx = \ln|x| + C$
- $\int e^x dx = e^x + C$
- $\int \sin x dx = -\cos x + C$
- $\int \cos x dx = \sin x + C$
- $\int \sec^2 x dx = \tan x + C$
- $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
- $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$

Differential Equations

- First-order linear: $\frac{dy}{dx} + Py = Q$
- Integration factor: $\mu = e^{\int P dx}$
- Solution: $y = \frac{1}{\mu}\left[\int \mu Q dx + C\right]$
- Variable separable: $\frac{dy}{dx} = f(x)g(y) \rightarrow \frac{dy}{g(y)} = f(x)dx$

Complex Numbers

- $i^2 = -1$, $i^3 = -i$, $i^4 = 1$
- Modulus: $|a + bi| = \sqrt{a^2 + b^2}$
- Argument: $\arg(a + bi) = \tan^{-1}\left(\frac{b}{a}\right)$
- Polar form: $z = r(\cos\theta + i\sin\theta)$
- De Moivre's theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

Problem-Solving Strategies

Matrix Problems

1. **Always check dimensions** before performing operations
2. **For matrix equations**: Use inverse method $X = A^{-1}B$
3. **For transpose properties**: Use $(AB)^T = B^T A^T$
4. **For matrix powers**: Calculate step by step, look for patterns

Differentiation Problems

1. **Identify the type**: Product, quotient, chain rule, or implicit
2. **For complex functions**: Break down using appropriate rules
3. **For applications**: Remember $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$
4. **For maxima/minima**: Find critical points where $f'(x) = 0$

Integration Problems

1. **Recognize standard forms** first
2. **For substitution**: Look for $f'(x)$ when $f(x)$ appears
3. **For integration by parts**: Choose u as LIATE (Log, Inverse trig, Algebraic, Trig, Exponential)
4. **For definite integrals**: Use fundamental theorem or properties

Differential Equations

1. **Identify the type**: Linear, separable, or exact
2. **For linear equations**: Find integration factor systematically
3. **For separable equations**: Separate variables completely before integrating
4. **Always check initial conditions** if given

Complex Numbers

1. **For operations**: Convert to $a + bi$ form first
2. **For polar form**: Calculate modulus and argument carefully
3. **For powers**: Use De Moivre's theorem
4. **For division**: Multiply by conjugate of denominator

Common Mistakes to Avoid

Matrix Operations

- ❌ **Don't assume** $AB = BA$ (matrix multiplication is not commutative)
- ❌ **Don't forget** to check if matrices can be multiplied (inner dimensions must match)

- ❌ **Don't confuse** transpose with inverse

Differentiation

- ❌ **Don't forget** the chain rule for composite functions
- ❌ **Don't mix up** $\frac{d}{dx}(\sin x) = \cos x$ and $\frac{d}{dx}(\cos x) = -\sin x$
- ❌ **Don't forget** to use product rule when multiplying functions

Integration

- ❌ **Don't forget** the constant of integration $+C$
- ❌ **Don't confuse** indefinite and definite integrals
- ❌ **Don't forget** to substitute limits properly in definite integrals

Complex Numbers

- ❌ **Don't forget** $i^2 = -1$ when expanding
- ❌ **Don't confuse** modulus with real part
- ❌ **Don't forget** to rationalize denominators with complex numbers

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Exam Tips

Time Management

- **Spend 2-3 minutes** reading the entire paper first
- **Attempt easier questions first** to build confidence
- **Reserve 15 minutes** at the end for review

Writing Strategy

- **Show all steps clearly** - partial marks are often awarded
- **Draw diagrams where helpful** - especially for geometry problems
- **Write final answers clearly** and box them if possible

Calculation Tips

- **Double-check arithmetic** - many marks are lost due to calculation errors
- **Use calculator efficiently** but don't become dependent on it
- **Cross-verify answers** using different methods when possible

Question Selection

- **In OR questions**, choose the one you're most confident about

- **Don't spend too much time** on any single question
- **If stuck**, move on and return later with fresh perspective

Good luck with your exam preparation!